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THEORY OF RAILWAY TURNOUTS.

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1. In *Trautwine's Pocket Book* (page 401, edition of 1883) it is stated that the *modern practice* of laying out turnouts is to curve the switch rails so as to form part of the turnout curve.

The object of this article is to show that the *modern theory* of locating turnouts depends on a single, simple and exact proposition. (See Art. 4.)

2. Fig. 1 represents a turnout from a straight track. A J and A′J′ are the two rails of the main line; H and H′ are the points from which the switch rails BH and B′H′ spring, when actuated by the switch lever at S. The distance B*b* is called the throw of switch, and (for the ordinary stub switch here considered) is usually 5 ins. for a 4′8½″ gauge.

It should be borne in mind that the switch rail, HB, when thrown over to the turnout, is not straight from H to *b*, but curved somewhat in the form of an elastic curve. For purposes of calculation, however, it is assumed to be a portion of a simple circular curve extending from H to F. In order to satisfy this condition, B*b*, or the throw of switch, must equal the tangent offset to the curve HF, corresponding to a length of tangent HB; and, as B*b* and the degree of curve HF are usually fixed, HB must be calculated accordingly. The difference HJ, between the length of switch rail and the total length of the rail, is spiked down to the ties.

At F, the intersection of the turnout curve with the main track, a frog is placed.

Frogs are usually designated by numbers which express the ratio $\frac{FC}{AB}$, Fig. 2.

It is sometimes difficult to determine the point F exactly, owing to the rounding off shown at S. In such cases the frog number may be found from the ratio $\frac{CG}{AB+DE}$; for, by similar triangles,

$$FG : FC :: DE : AB$$

$$\therefore \quad FG+FC : FC :: DE+AB : AB$$

or
$$\frac{CG}{AB+DE} = \frac{FC}{AB} \quad \text{Q.E.D.}$$

Another common way of determining the frog number, is to slide a rule along the frog until it measures any whole number of inches across, (say 4″, as shown in Fig. 2). Mark the place and then slide the rule along until the breadth becomes 5 inches. The distance *d*, between the two cross lines in inches, will be the number of the frog.

If the frog number be known the exact point F or intersection of the gauge sides of the rails may be readily found by determining the point p, Fig. 2, when the

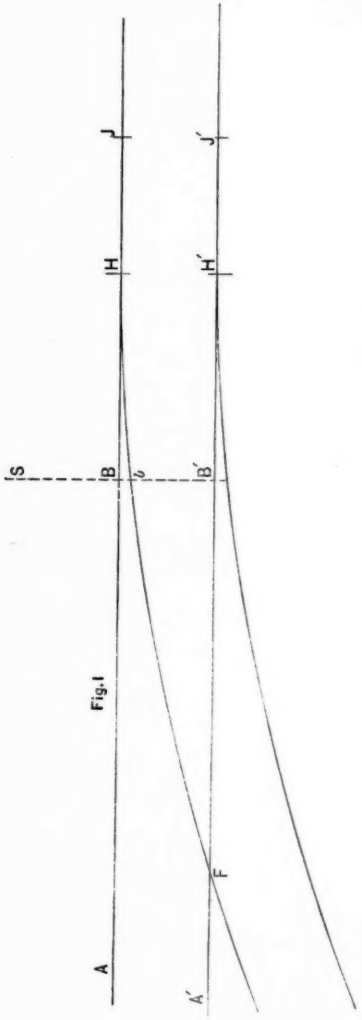


Fig. 1

distance across is exactly 1 inch. Then measure from this point to F as many inches as there are units in the number of the frog.

To find the frog angle from the number of the frog.

In the triangle AFC, Fig. 2,

$$\frac{FC}{AC} = \cot.\tfrac{1}{2}\,AFB.$$

Put n = number of frog, f = frog angle then

$$n = \frac{FC}{AB} = \frac{FC}{2AC} = \tfrac{1}{2}\cot.\tfrac{1}{2}f\,. \quad (1.)$$

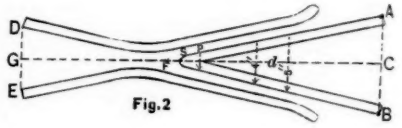


Fig. 2

3. THEOREM I.—*The angle included between lines drawn from the point of frog to the heels of the two switch rails, is equal to one-half the frog angle*, or HFH' in figures 3, 4 and 5 equals $\frac{1}{2}f$.

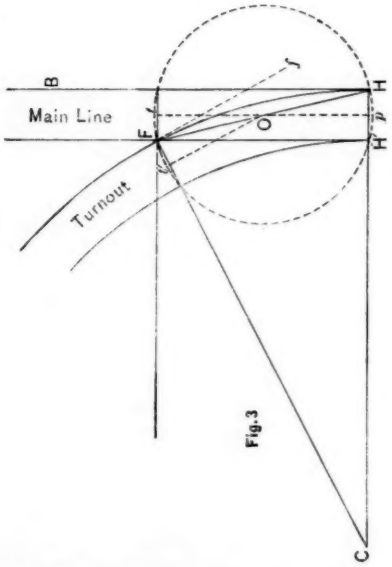


Fig. 3

CASE 1.—*Turnout from a tangent.*

In Fig. 3 the frog angle fFH = FCH, since Ff is perpendicular to FC and FH' to CH.

Also H'FH = FHB = the deflection angle of turnout curve, and hence is $\frac{1}{2}$ the intersection angle FCH.

$\therefore$ H'FH = $\frac{1}{2}$ H'Ff Q. E. D.

CASE 2. *Turnout from inside of curve.* Fig. 4.

In Fig. 4, C and C′ are the centers of the main line and turnout respectively, CFC′=frog ang. fFf', since CF and C′F are perp. to Ff and Ff'.

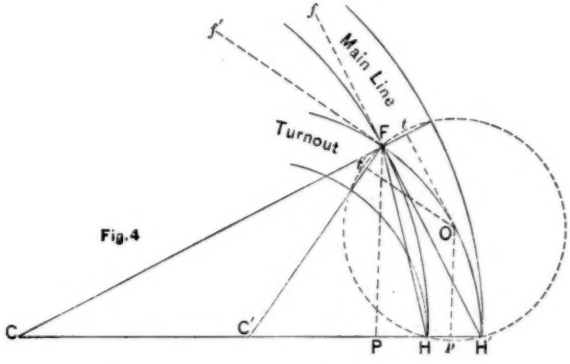


Fig. 4

Let fall the perp. FP upon CC′ then

H′FP=½ FC′H
H FP=½ FC H
Subtract HFH′=½ CFC′ Q. E. D.

CASE 3. *Turnout from the outside of a curve.* Fig. 5.

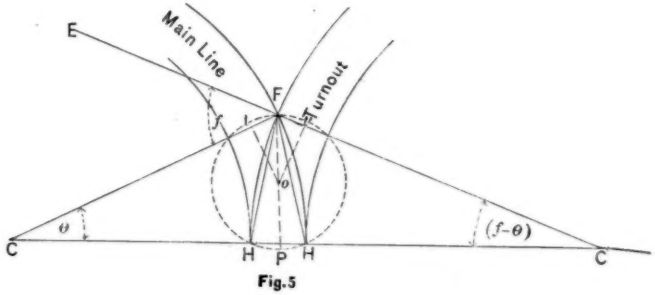


Fig. 5

Let fall the perp. Fp upon CC′; then

H′FP=½ FC′H′
H FP=½ FCH
adding HFH′=½ EFC Q. E. D.

COROLLARY.—Since the angle HFH′ is constant, the gauge and frog angle remaining the same, it is evident that the locus of the point of frog is the circumference of a circle passing through HH′ and F as shown by dotted circles in Figs. 3, 4 and 5. The diameter of this

$$\text{circle} = \text{FH (Fig. 3.)} = \frac{\text{gauge}}{\sin. \frac{1}{2} f} \quad . \quad (2.)$$

4. THEOREM II.—*Those parts of the center lines of the main and turnout tracks extending from the heel of switch to the point of frog have a common intersection point* (O Fig. 6) *and their tangents* (Op, Ot *and* Ot' Fig. 6) *are equal to the product of the gauge into the frog number.*

In Fig. 6, O is the center of the circle passing through the points F, H, H.′ C and C′ are the centers of the main line and turnout.

Produce CF, and take Fm=H′H=gauge. The triangles OFC and OH′C equal, hence, <OFC= <OH′C and <OH′H=OFm. Hence the triangles OFm and OH′H have two sides and the included angle of one equal to corresponding parts of the other.

Hence, $Om = OH$, and m is a point on the circumference HH'F. The point n on Fc' so taken that $Fn = HH'$ is also on this circumference. Therefore lines drawn from O to the middle points of the chords Fm, Fn and HH' at t, t' and p will be equal to each other, and also perp. to the

RULE.—*The radius of a turnout from a tangent, equals twice the gauge into the square of the frog number.*

H'F (Fig. 3) $= pt = 2gn$, since

$pt = pO + Ot$,

also $t'Ot = f$ = frog angle.

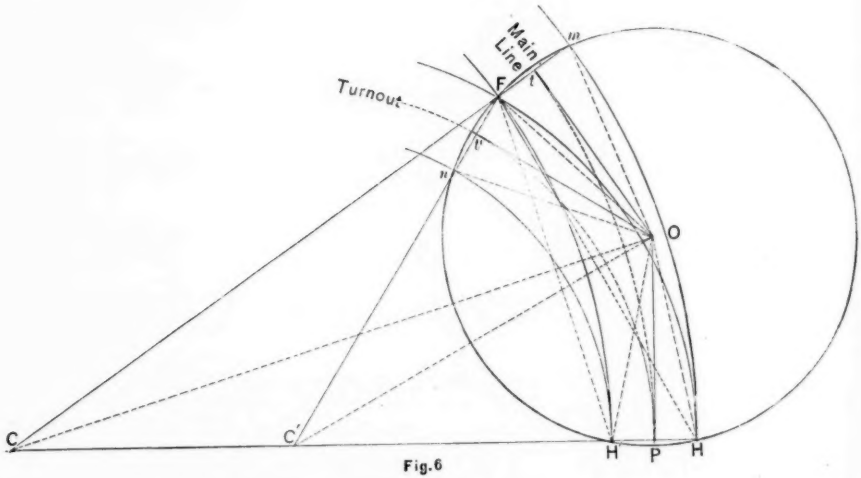


Fig.6

radii (Ct $C't'$ and Cp) of the center lines of the main and side tracks. Therefore Ot Ot' and Op are the tangents of the curves pt and pt', and O is the common intersection point.

By theorem I, ang. H'FH equals $\frac{1}{2}$ the frog angle; but $H'OH = 2H'FH$.

$\therefore$ $H'OH = f$ = frog angle.

$$\therefore \frac{Op}{H'H} = \text{no. of frog} = n,$$

$$\text{or } \frac{\text{Tangent}}{\text{gauge}} = n$$

$\therefore$ Tangent $= g \times n$ (3) Q. E. D.

5. PROBLEM.—*Given: the frog number n, and the gauge of the track g, to find the radius of curvature of a tangent turnout.* (Cp, Fig. 3.)

By Theorum II, the tangents of the turnout curve $= g \times n$ ($= Op = Ot = Ot'$). But tan. of curve $= R \times \tan.\frac{1}{2}f$ ($f = \angle t'Ot$)

$$\therefore g \times n = R \times \tan.\tfrac{1}{2}f = \frac{R}{\cot.\frac{1}{2}f} = \frac{R}{2n} \text{ by (1).}$$

and $R = 2gn^2$ . . . . (4).

Hence the

This affords a most convenient way of staking out turnouts by setting the transit at the intersection point O and centering t and t' at a distance from $O = g \times n$.

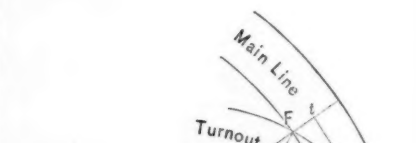


Fig.7

6. *Given: a main track on a 4° curve* (R = 1432.7), *and a number 9 frog* (*ang.* = 6°22'), *to locate a turnout from the inside of the curve.* See Fig. 7.

Put θ=FCH—the total angle of the 4° curve from heel of switch to point of frog.

g=gauge=4.71 ft.

n=frog No.=9.

By Theorem II,

tangents=$g\times n$=4.24 ft.

But tangent=radius$\times\tan.\frac{1}{2}\theta$

$$\therefore\ 42.4=1432.7\times\tan.\tfrac{1}{2}\theta$$

$$\theta=3°24'$$

Hence, $\theta+f$=intersection angle of the turnout curve=9°46′.

To find r, the radius of the turnout curve,

$$Ot'=gn=r\times\tan.\tfrac{1}{2}(\theta+f)$$

$$r=\frac{gn}{\tan.\frac{1}{2}(\theta+f)}=\frac{42\,4}{.08544}=496.5$$

corresponding to a 11°34′ curve.

These values of θ and $\theta+f$ may be found more quickly by using the table of tangents to a 1° curve in Shunks' *Field Engineer*. Thus $42.4\times4°=169.6$, the corresponding tangent of a 1° curve. Referring to the table we find that an angle of 3°24′ corresponds to a tangent of 170 ft., which is sufficiently exact.

It may be here remarked that if a 50 ft. chord be used, Shunks table of functions for a 1° curve is practically exact, even when applied to sharp curves, as for this chord length the radii are very nearly in the inverse ratio of the degree of curve.

The turnout, Fig. 7, may be located on the ground, by setting the transit at p and running the 4° and 11°34′ curves to t and t'; or lay off the tangent pO, then from O set off $Ot(=g\times n)$ making an angle with Op=3°24′ and $Ot'(=g\times n)$ by turning an additional angle tOt'=6°22′.

Similarly, the conditions of a turnout from the outside of a curve may be determined. In this case, however, the intersection angle of the turnout will equal the difference of θ and f instead of their sum.

7. In practice calculations may be much shortened by using convenient approximations, thus let $2gn$ (the sum of the tangents) be taken as the length of both main line and turnout. This assumption is nearly true for ordinary curves less than 100 ft. long. Let θ = intersection angle of the main line, D its degree of curve, and f the frog angle; then,

$$\theta : D :: 2gn : 100$$

or
$$D=\frac{50\theta}{gn} \quad . \ . \ . \ . \ (5).$$

If the turnout be from the inside of the main line, the intersection angle will be $f+\theta$. (See Art. 6.)

Put D′=degree of curve of turnout; then
$$f+\theta : D' :: 2gn : 100$$

or
$$D'=\frac{50(f+\theta)}{gn} \quad . \ . \ (6).$$

Subtracting (5) from (6)

$$D'-D=\frac{50f}{gn} \text{ or } D'=D+\frac{50f}{gn} \quad . \ (7).$$

If the turnout be from a tangent, make θ and D=zero, and equation (6) becomes

$$D'=\frac{50f}{gn} \quad . \ . \ . \ . \ (8).$$

It is seen by comparing (7) and (8) that *the degree of curve of a turnout from the inside of a curve, equals the degree of curve of the main line, plus the degree of curve of a turnout from tangent, the gauge and frog angle remaining the same.*

Example. Given: g=4.71, n=9, f=6°22′; to find D′ for a turnout from tangent, and also from the inside of a 4° curve.

By equation (8)

$$D'=\frac{50\times6°22'}{4.71\times9}=\frac{19100'}{42.4}=450'=7°30'$$

which is the degree of curve of a turnout from a tangent.

Again from equation (7)

$$D'=4°+7°30'=11°30'.$$

Compare with the result in Art. 6.

8. For a turnout from the outside of a curve, the intersection angle of the main line being θ, that of the turnout will be $f-\theta$. See Fig. 5.

Let D′=degree of turnout curve, and D degree of curve main line, then

$$f-\theta : D' :: 2gn : 100$$

$$D'=\frac{50(f-\theta)}{gn} \quad . \ . \ . \ (9).$$

Adding (5) and (9)

$$D+D'=\frac{50f}{gn} \ . \ D'=\frac{50f}{ng}-D \ . \ (10).$$

Hence the degree of curve of a turnout from the outside of a curved main line equals the difference between the degrees of curve of the main line and the corresponding tangent turnout.

EXAMPLE, Given: $g = 4.71$, $n = 9$, $f = 6°22'$, to find D′ for a turnout from the outside of a 4° curve.

By eqs. (8) and (10) $D' = 7°30' - 4° = 3°30'$. In this case the main line and turnout will curve in opposite directions.

EXAMPLE, Given: $g = 4.71$, $n = 9$, $f = 6°22'$, to find the degree of curve of a turnout from the outside of a 10° curve.

Here $D' = 7°30' - 10 = -2°30'$, and the main line and turnout will curve in the same direction.

Then, degree of curve $ca = 7°30' + 3° = 10°30'$. If ba and ca each equal 50 ft., then ang. $dab = \frac{3°}{4}$ (da being the common tangent at a).

$$\text{ang. } cad = \frac{10°30'}{4}$$

$$\therefore \text{ang. } cab = \frac{7°30'}{4} = 1°52\tfrac{1}{2}' \text{ as before.}$$

For a turnout from the outside of a curve, the sum of the deflection angles of the main line and turnout, for any given distance, equals the deflection angle of the turnout from tangent for the same distance.

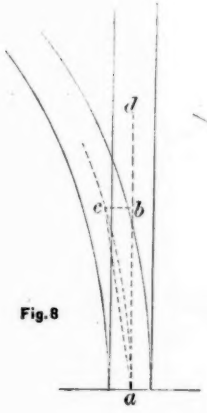


Fig. 8

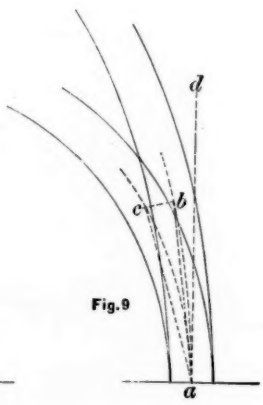


Fig. 9

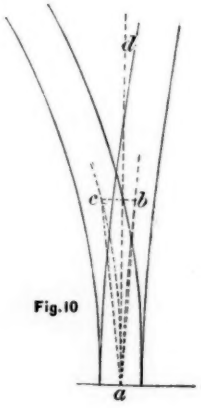


Fig. 10

9. Since [eq. (7)], for a turnout from the inside of a curve, the difference of the degrees of curvature of main line and turnout is constant, and equal to the degree of curve of a turnout from tangent; it follows, that the difference of the deflection angles of the main line and turnout, for any given distance, will equal the deflection angle, for the same distance, of the turnout from a tangent.

Thus let Fig. 8 represent a turnout from tangent, $g = 4.71$, $n = 9$. The degree of turnout curve will be 7°30′. Let a be the heel of switch, and take ab and ac each 50 ft. Then the angle $cab = \frac{7°30'}{4}$ $= 1°52\frac{1}{2}'$. Fig. 9 represents a turnout from the inside of a 3° curve, g and n remaining the same.

Thus, Fig. 10, if ab be a 3° curve ac will be a 4°30′; hence

$$\text{ang. } dab = \frac{3°}{4} \text{ (ab being 50 ft.)}$$

$$\text{ang. } dac = \frac{4°30'}{4}$$

$$\text{adding, } cab = \frac{7°30'}{4} = 1°52\tfrac{1}{2}' \text{ as before.}$$

Since in figures 8, 9 and 10, the angle

$$bac = 1°52\tfrac{1}{2}'$$

and $$ac = ab = 50 \text{ ft.}$$

$$cb = 2 \sin. \frac{1°52\frac{1}{2}'}{2} \times 50;$$

or, in general, $cb = 2 \sin. \frac{1}{2} bac \times ac$. (11).

A table based on (11) would afford a ready means of laying out turnouts. Assuming convenient values for ab the corresponding values of cb are calculated for the proper degree of curve (ac Fig. 8). In practice a cord or tape should be held at a (the heel of switch) and the proposed diatance laid off to b, then the tape is swung around a as a center, until the corresponding value of cb is obtained.

The distances cb may be set off from either rail of the main line to the corresponding rail of the turnout, but this presupposes that the main line is laid and approximately lined.

This method of putting in turnouts has two advantages:

1st. It is uniform in its application to tangents and curves of all kinds, but one table being required (g and n remaining the same).

2d. Since turnouts are laid on switch ties, it is of but slight importance that the main track be in perfect line before putting in the turnout, for the off-sets determine the correct position of one rail with reference to the other, so that when one is brought to line the other must necessarily follow.

By modifying the value of the gauge in the formulæ a short tangent may be obtained through the frog, which is the customary practice.

10. Switch Rails. — It was before stated that the length of switch rail was usually so taken that the throw of switch, Bb, Fig. 1, is the corresponding offset from a tangent HB to the turnout curve at b.

Assuming the tangent offsets to vary as the squares of their distances from the tangent point, we have, Fig. 1,

$$HH' : Bb :: \overline{HF}^2 : \overline{HB}^2.$$

Whence

$$HB = H'F\sqrt{\frac{Bb}{HH'}};$$

but $H'F = 2gn$. (Theorem II.)

and $HH' = g$. $Bb = t$ = throw.

$$\therefore \quad HB = \sqrt{4gn^2 \times t} = \sqrt{2r \times t} \quad . \quad (12).$$

since $2gn^2 = r$.

Rule. — *To find the length of switch rail for a tangent turnout, multiply twice the radius of the turnout by the throw of switch and extract the square root of the product.*

By (12) $HB = \sqrt{4gn^2 \times t}$,

or $HB = 2n\sqrt{gt}$,

Hence the length of switch rail is directly proportional to the frog number, the gauge and throw remaining the same. Thus for a 10 frog, $g = 4.71$ $t = 5$ in. $= .417$ ft.

$$\therefore \quad HB = \sqrt{4 \times 471. \times .417} = 28 \text{ ft.}$$

For a 9 frog $HB = \frac{28 \times 9}{10} = 25.2$ ft.

For an 8 frog $HB = \frac{28 \times 8}{10} 22.4$ ft.

For a 10 frog, the gauge being 3 feet, $t = .333$ ft.

$$HB = \sqrt{1200 \times \tfrac{1}{3}} = 20 \text{ ft.}$$

For an 8 frog $HB = \frac{8 \times 20}{10} = 16$ ft.

If the turnout be from a curve the length of switch rail will not be changed, for it has been shown in Art. 9 that ab being constant the values of bc are the same in Figs. 8, 9 and 10; hence if bc be taken as the throw in the three figures, the values of ab, or the length of switch rail, will be the same for the turnouts from curves, Figs. 9 and 10, as for the tangent turnout, Fig. 8.

11. It has been assumed that the switch rail springs to a circular curve of the same radius as the rest of the turnout.

Putting t = throw.
l = length of switch rail.
S = switch angle;

or the total curvature of the switch rail from heel of switch D, Fig. 11, to the head block A, then

$$\tan. S = \frac{t}{\frac{1}{2}l} = \frac{AC}{AB}, \text{ Fig. 11.} \quad . \quad (13).$$

If the rail be influenced by the switch lever alone, neglecting the friction on the ties, the curve assumed will be the elastic curve.

Hence $\tan. S = \frac{t}{\frac{2}{3}l'} = \frac{AC}{AB'}$, Fig. 12 . (14).

Equating (13) and (14),

$$\frac{t}{\frac{1}{2}l} = \frac{t}{\frac{2}{3}l'},$$

$$\therefore \quad l' = \tfrac{3}{4}l \quad . \quad . \quad . \quad (15).$$

Thus l for a switch rail sprung in a circular arc, when $g=4.71$, and $n=9$, will be 25.2 ft.

If, however, the switch rail be bent to an elastic curve, the switch angle being the same, then Eq. (15),

$$l'=\tfrac{3}{4}\times 25.2 \text{ ft.}=18.9 \text{ ft.}$$

The true curve lies somewhere between

$D'+D''$=degree of curve of turnout from tangent corresponding to frog F_m (19), (number=N).

Comparing (18) and (19) it is seen that F_m will be the proper frog for a tangent turnout whose degree of curve is double that of a similar turnout for the frogs F and F'.

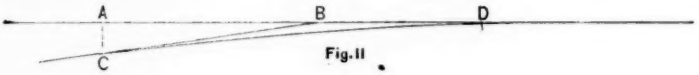


Fig. 11

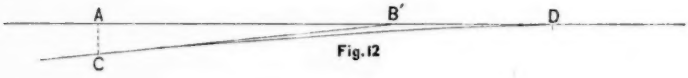


Fig. 12

these extremes, and, in actual practice, the switch rail is spiked up until the proper switch angle is obtained. Hence all calculations may be made for simple circular curves, the length of switch rail being adjusted experimentally to fit the rest of the turnout.

12. Double Turnouts.

A double turnout from a tangent is shown in Fig. 13.

Here three frogs are used, two of which F and F' are alike, and the angle of the middle frog F_m should be so taken that the turnout curves HF_mF, and $H'F_mF'$ will be simple circular curves.

Let D =degree of curve of main line.

D' =degree of curve of turnout to right.

D''=degree of curve of turnout to left.

Then by Art. 7, Eq. (7):

$$D''=D+\text{constant} \quad . \quad . \quad (16).$$

By Eq. (10),

$$D'=-D+\text{constant} \quad . \quad . \quad (17).$$

Adding

$$D''+D'=2 \text{ constant} \quad . \quad . \quad (18).$$

The constant is the degree of curve of a turnout from a tangent with the given frogs F or F'.

Now, if D'' be considered as a turnout from the outside of D' as a main line, then by (10),

Putting R=radius of tangent turnout for the frog F_m,

and r=radius of tangent turnout for the frog F.

and assuming that the radii are inversely

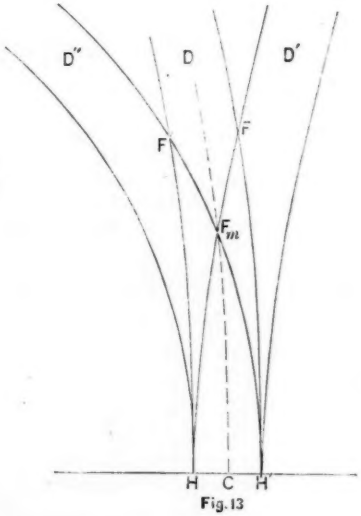


Fig. 13

proportional to the degrees of curvature, then

$$R=\tfrac{1}{2}r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$$

Put n=No. of frogs F and F'.

N= " " frog F_m.

then by (4),

$$R=2gN^2 \quad . \quad . \quad . \quad . \quad . \quad (21).$$

also $$r=2gn_2 \quad . \quad . \quad . \quad . \quad . \quad (22).$$

Comparing (20), (21) and (22),

$$gn^2=2gN^2,$$

or $$N=\frac{n}{\sqrt{2}}=.707n \quad . \quad . \quad . \quad (23)$$

The frogs F and F′ may be located as though they were single frogs.

From the foregoing it will be remarked that a three-throw set of frogs consists of two frogs of like number (n), and a third or crotch frog, whose number $N=\frac{7}{10}n$ nearly; and, further, that this set is just as applicable to a curved main line as to a straight one.

13. Side Tracks.—*Given: a turnout from tangent with a frog angle* f (*No.*=n), *and the perpendicular distance* (p) *between the center lines of the main and side tracks, to join the turnout with the side track, by a curve reversing at the frog point.*

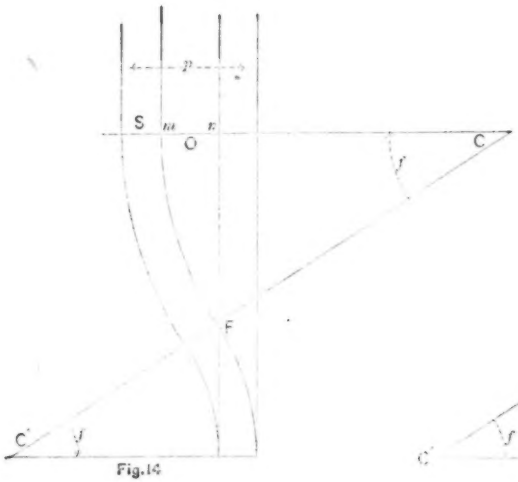


Fig. 14

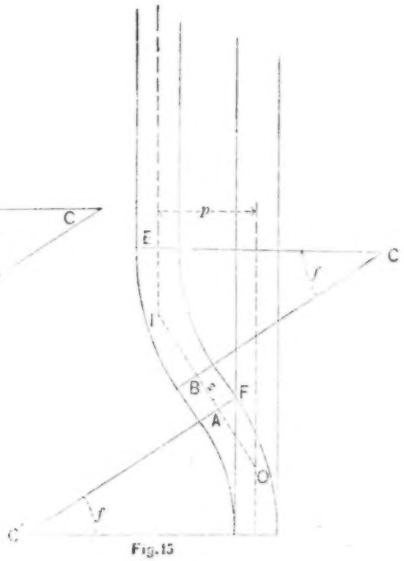


Fig. 15

To locate F_m the center line CF_m, Fig. 13, is assumed to represent the gauge side of a rail. A glance at the figure will show that the point F_m is the proper location of a frog whose number is 2N, the gauge of track being $\frac{1}{2}g$.

Hence, approximately,

$$CF_m=2\times\tfrac{1}{2}g\times 2N=2gN;$$

but (23) $$N=.707n,$$

$$\therefore \quad CF_m=.707\,(2gn) \quad . \quad . \quad . \quad (24).$$

Or the point of crotch frog is located at a distance from the heel of switch equal to $\frac{7}{10}$ the distance of the main frog from the same point, the distances being measured along the center line of the main track.

An inspection of Fig. 14 will show that $mn=(p-g)$ may be taken as the gauge of a turnout, m being the heel of switch, F the frog, and mF the outside rail of the imaginary turnout.

By (4)

$$R=2gn^2$$

$$\therefore \quad CO=2(p-g)n^2,$$

and the required radius:

$$CS=CO+\tfrac{1}{2}p=2(p-g)n^2+\tfrac{1}{2}p \quad . \quad . \quad (25).$$

The total angle of the curve will equal the frog angle.

It is customary, however, to insert a short tangent between the frog and the

P. C. of the connecting curve, as AB, Fig. 15.

By Theorem II. $OA=gn$;

also, $$OI=\frac{p}{\sin.f}.$$

But $BA=l$=length of tangent.
But $BI=t$=tangent of curve BE.

Then $$t=\frac{p}{\sin.f}.-(gn+l),$$

and since the angle at $I=f$,

$$CB=R=\frac{\frac{p}{\sin.F}-(gn+l)}{\tan.\frac{1}{2}f}$$

$$=\left(\frac{p}{\sin.f}-(gn+l)\right)\cot.\tfrac{1}{2}f.$$

But by (1) $\cot.\frac{1}{2}f=2n.$

$$\therefore \quad R=\left(\frac{p}{\sin.f}-(gn+l)\right)2n \quad . \quad . \quad (26).$$

or since $$\frac{p}{\sin.f}=pn \text{ nearly},$$

$$R=[(p-g)n-l]2n \quad . \quad (27).$$

14. *Given: a turnout from a curved main line, to connect the turnout curve with a side track, by a simple curve beginning at the point of frog.*

Put p=distance between centers of main and side tracks, R=radius of main line, r=radius of the required connecting curve.

Case I.—*The siding outside the main track.* Fig, 16.

Let mF, as before, be taken as the outer rail of a turnout curve, m being the heel of switch and F the point of frog, $mn=(p-g)$ being the gauge.

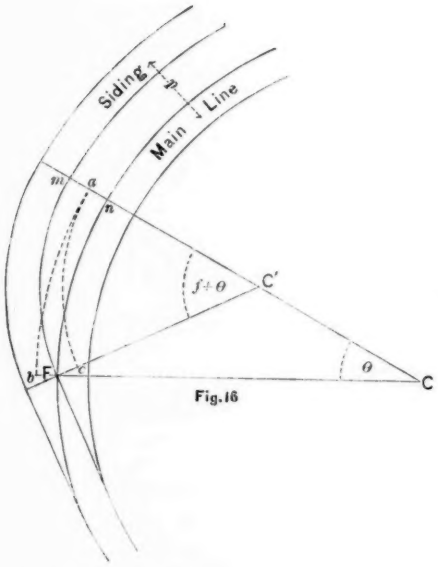


Fig. 16

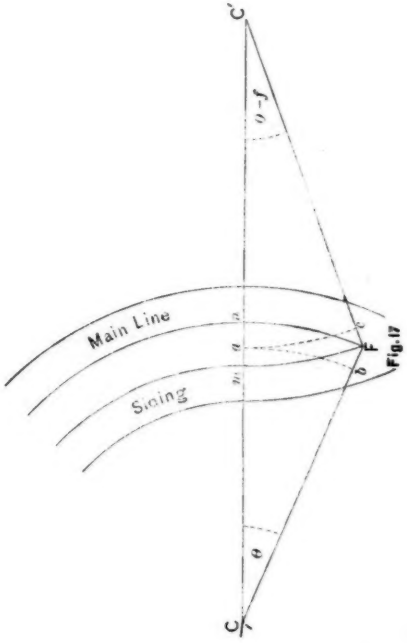


Fig. 17

Then, by theorem II., the tangents of the arcs ab and ac are equal to $(p-g)n$;

also $$\tan.\tfrac{1}{2}\theta=\frac{\text{tangent}}{\text{radius}}$$

$$\therefore \quad \tan.\tfrac{1}{2}\theta=\frac{(p-g)n}{R+\frac{1}{2}p} \quad . \quad . \quad . \quad (28).$$

Also $$\tan.\tfrac{1}{2}(f+\theta)=\frac{(p-g)n}{C'a},$$

where $C'a=r-\frac{1}{2}p$.

$$\therefore \quad r=\frac{(p-g)n}{\tan.\frac{1}{2}(f+\theta)}+\tfrac{1}{2}p \quad . \quad (29).$$

Case II.—*The siding inside of the main line.* Fig. 17.

As before, take mF as the outer rail of a turnout curve, m being the heel of switch and $p-g$ the gauge.

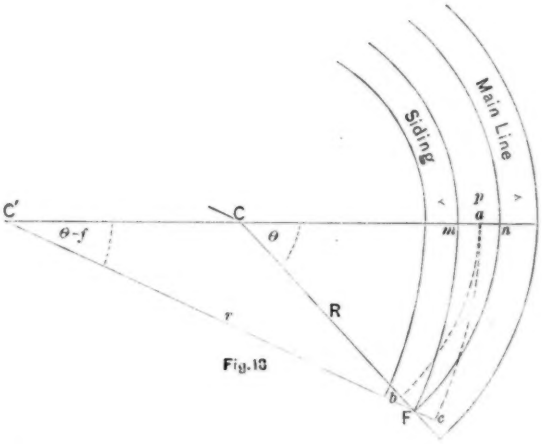


Fig. 18

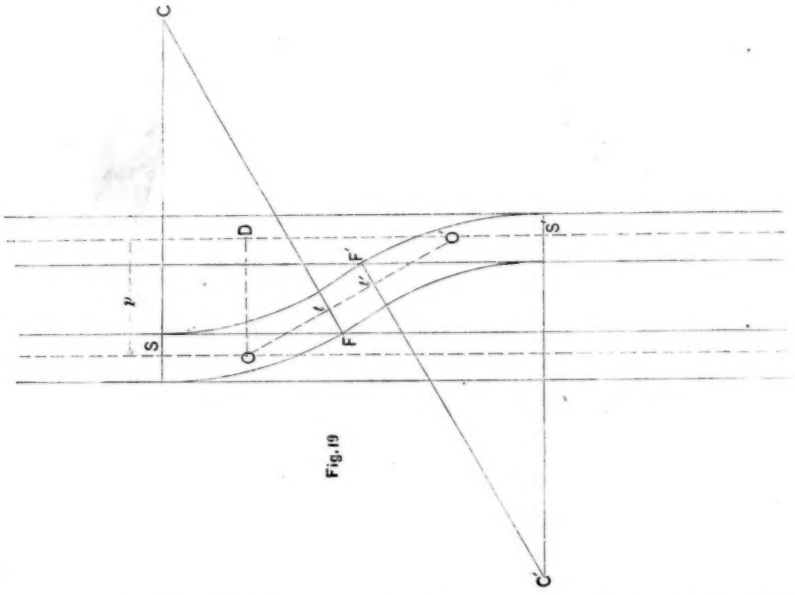


Fig. 19

$$\tan.\tfrac{1}{2}\theta = \frac{\text{tangent}}{\text{radius}} = \frac{(p-g)n}{R-\frac{1}{2}p} \quad . \quad (30).$$

Also $$\tan.\tfrac{1}{2}(\theta-f) = \frac{(p-g)n}{C'a}.$$

$$\therefore\ C'a+\tfrac{1}{2}p = r = \frac{(p-g)n}{\tan.\frac{1}{2}(\theta-f)} + \tfrac{1}{2}p \quad . \quad (31).$$

In Fig. 18 the siding is on the inside of the main line, but θ is greater than f, and the centers of main line and connecting curve are on the same side of the main line. The preceding formulæ then become

$$\tan.\tfrac{1}{2}\theta = \frac{\text{tangent}}{\text{radius}} = \frac{(p-g)n}{R-\frac{1}{2}p} \quad . \quad (32).$$

$$\tan.\tfrac{1}{2}(\theta-f')=\frac{(p-g)n}{C'a}$$

$$\therefore\ C'a-\tfrac{1}{2}p=r=\frac{(p-g)n}{\tan.\frac{1}{2}(\theta-f')}-\tfrac{1}{2}p \quad (33).$$

15. The Link.

A link, or cross over track, is represented in Fig. 19, the parallel tracks being straight. The two frogs F and F′ are alike, and it is required to determine the distance tt'.

Taking g=gauge, n=No. of frog, and p=perp. distance between parallel track centers. then

ang. OO′D=frog angle f

$$\therefore\ OO'=\frac{p}{\sin.f}$$

also $O't'=Ot=g\times n$

$$\therefore\ tt'=\frac{p}{\sin f}-2gn \quad . \quad . \quad (34).$$

or approximately $OO'=pn$

$$O't'=Ot=gn$$

$$\therefore\ tt'=pn-2gn=(p-2g)n \quad . \quad . \quad (35)$$

EXPLOSIVES.

By M. BERTHELOT, Paris.

Translated by M. BENJAMIN, Ph. B., F. C. S.

I.

General Observations in the Force of Explosives.

1. The force of an explosive may be understood in two ways according to the different senses in which the word is applied, that is, it may be considered either as the pressure developed or as the work accomplished. It frequently happens that the word force is used to represent the *pressure* resulting from the explosive, or (to put it more definitely) that produced by the gas arising from the decomposition.

It is this which is the cause of the bursting of hollow projectiles and the breaking down of walls in mines.

But this definition is not complete, for the reason that hydraulic pressure would effect similar results without producing any notable ultimate effects, whereas certain mechanical results follow as the result of explosives, such as the noise, or the extended fracture of rocks, the projection of balls, of bullets, and of fragments from hollow shells shattered by the explosion.

2. The following detailed list includes the principal applications of explosives in industrial or military arts.

First.—The bursting of hollow projectiles by black powder or its substitutes.

Second.—The breaking of masses of cast or wrought iron, such as bears, which accumulate below the tap hole of cupolas, or which form in the crucibles of blast furnaces, causing the suspension of all work. Black powder is almost entirely without action on such material, and it becomes necessary to use some stronger agents, such as nitro-glycerine, or dynamite, or even compressed gun-cotton, in order to break into pieces the cast iron or wrought iron.

Third.—The destruction of metallic bridges by twisting, tearing, or otherwise removing them from their location, to prevent their use in times of war, with the utter ruin of their fragments either on land or under the water. so that the bridge cannot be reconstructed.

Fourth.—The breaking, rupturing and piercing of rails and metallic plates, such as sheeting and the like.

Fifth.—The bursting or putting out of service of pieces of ordnance, steel, cast iron or bronze, either by exploding them with dynamite placed in their boxes, or else by similar treatment externally so as to destroy the trunnions.

Sixth.—The blasting of rocks by means of dynamite, gun-cotton, or any of the various forms of black powder (gun powder, mine powder, etc.).

This blasting may be for the simple dislocation of the rocks, or else for their reduction into smaller or larger pieces, which remain *in situ* or are trimmed and piled in heaps for some industrial usage, or which may be employed for

some military operation. Finally, it is possible to pulverize the rocks into powder or into very small pieces when it is desired to dig a hole or an opening in the ground. The differences in rocks, on account of their hardness or tenacity, their aquiferous or fissure-like character demand a great diversity in the use of explosives, which are necessary to produce any given result.

Some very interesting applications of both dynamite and gun-cotton have resulted from submarine blasting and by their employment, it becomes possible to make constructions, which thus far have been deemed impracticable.

Seventh.—The destruction and excavation of clay banks or earth works by dynamite.

The digging of chambers and passages in clay or earth.

Eighth.—The demolition of all kinds of masonry, the destruction of bridges, tunnels, constructions of all sorts, galleries in mines, etc.

Ninth.—The breaking of ice and the removal of icicles by extensive displacement of the material; for this variety of work dynamite is especially adapted.

Tenth.—The breaking of wood by splitting, cutting or tearing, such as the removing of standing timber by using dynamite in land clearing or in war; the breaking down or overthrowing of palisades; the demolition and breaking up of piles under water; the tearing up and breaking of buried stumps of trees.

Eleventh.—The destruction of floating vessels, the breaking up of stranded ships or submarine wrecks.

Twelfth.—The destruction of torpedoes, of submarine or subterranean mines from a distance.

Thirteenth.—The projection of balls, bullets, shells, etc, from various weapons, guns, cannon, etc.

Fourteenth.—The projection of rockets by the composition of an internal charge of powder.

Fifteenth.—The ignition by primers or detonators, which determine explosions in the main body of gunpowder or dynamite.

We shall not at present refer to the pyrotechnic displays, that is to say, the use of powder on the producing agent of light and fireworks. The theory of these displays is an entirely different one from those which we are about to consider.

3. The different applications of explosives which we have just enumerated are produced equally by the pressure and the effect of these substances.

The *pressure* depends principally upon the nature of the gases formed, on their volume and on the temperature.

The *work* on the other hand is principally dependant upon the amount of heat given off, which is the criterion of the energy developed. In other words, the *maximum work* which an explosive substance is capable of producing is proportional to the quality of heat given off in consequence of the chemical decomposition of the explosive, this matter being taken at the existing pressure and temperature, and its theoretical products reduced to the same conditions.

4. Let Q be this quantity of heat, expressed in calories, the corresponding effect translated into kilogrammeters would be according to the mechanical equivale t of heat, 425 Q.

This figure expressed the *potential energy* of the explosive.

Without a doubt, this is a limit that is never reached in practice, but it is necessary to be familiar with it as the only absolute term for comparison.

5. The effective transformation of this energy into work depends upon the volume of gas, the temperature, and the law of expansion. The transformation is never complete; worse than this, only a portion of the work itself is utilized in the application. For instance, in guns, the work which communicates to the projectile its living force, is the only one which is taken into account; it represents the actual amount utilized, while the work used up at the expense of the walls of the weapon, and by the gases and the air projected, is lost. An important fraction of the energy always remains unused in the form of heat locked up in the gases, or else communicated to the projectile, to the gun, etc.

The calculation of the proper distribution of energy between the heating property so called, the mechanical result accomplished, the living force communicated, the vibrating movements of the ground and of the air, is a most complicated affair.

6. The following are some general ideas in this connection, ideas which we think it best to present at this juncture. In consideration of the work for which explosives are to be employed, they are distinguished as *strong powders* and *weak powders*, *rapid powders*, and *slow powders*.

7. *Strong and rapid powders.*—The materials whose chemical decomposition takes place very rapidly, such as mercury fulminate, produce principally the effect of crushing rocks *in situ*, or breaking the shells of hollow projectiles into a multitude of small fragments, the elasticity of the entire mass not having had time to come into play; they constitute what are called the *breaking powders*.

Furthermore, the living force of translation communicated to particles contiguous to the powder, becomes predominant in consequence of the sudden production of the enormous pressures which is particularly characteristic of this class of powders. However, its influence exerts itself in a special manner on the surrounding gases, the molecules of which find themselves thrown out all of a sudden with a rapidity very much greater than that of their actual change of place, which is, as is known, comparable to the exact rapidity of sound in gases; in consequence, the molecules of gas tend to accumulate, the one on top of the other, and to produce the effects of a shock, and even of rupture, which may be compared to those which result from the shock or the pressure of an extremely hard, solid body.

Such are the extreme effects produced by the almost instantaneous explosion of a breaking powder.

But if the decomposition is retarded a little, and if the potential energy is considerable (*strong powders*), the explosive has a tendency to produce a tearing or shearing in the lines of the least resistance, even of metals having the greatest resisting powers.

These results extend for some distance into the mass of compact substances of moderate tenacity; they are the effects of dislocation. They produce the results without projection, provided that the masses to which the movement is communicated are of sufficient size.

In the employment of strong and breaking powders, it is possible to suppress or lessen the tamping, the communication of the presure taking place by mere contact before the substances have time to be driven away by the compressed air.

It is thus, that a feeble charge of dynamite placed in the open air on a hewn stone, and covered by a simple sand-bag, is sufficient to break that stone into small pieces. A single cartridge containing 150 grains of dynamite (of 75 per cent.), will, in this way, break a block having a surface equal to 60—80 square decimeters and a thickness of 40 centimeters. The piece will be broken up according to the cracks which radiate from the center of the explosion, and is analagous to that which would be produced by the falling of an iron beam from a great height. In a word, the effect is that of a gigantic shock, and is extremely sharp. Besides this dynamite may be used to break a block according to a given design, as if by a wedge. All that is necessary is that a furrow shall be marked along the surface, with a center drill-hole into which the charge is placed.

It is in consequence of this means of propagating the pressure, that the depth of a blast-hole may be made much smaller when dynamite is used. This is not all. In a blast-hole, the effect of the layers and crevices in the rock do not seriously influence the action of such a powder, providing, however, that the layers or crevices are not directed towards the center of the shock. These powders are also greatly preferred for displacing fissured and aquiferous territory; they excel all others in tearing down a sand bank, or for enlarging a stony excavation. The conditions under which they act, are such that they may be employed equally as well for drilling an opening in the ground, of medium size, perpendicular to the surface, always in the direction of the least resistance, and without paying any attention to the breaking of rocks that may lay in their pathway.

With such powders, the effect of successive explosions in the same chamber are accumulative, that is to say, the fissures produced by the first shock are increased with the second; by taking advantage of this circumstance, it is possible to obtain pieces of much greater dimensions than would have been obtained by one single operation, using the entire quantity of dynamite.

These different characteristic properties showing the action of dynamite, lead us to regard it as the type of the strong and rapid powders.

8. *Strong and Slow Powders.*—Black powder or gunpowder is also a strong powder, although for equal weight, of considerable less power than dynamite; but at the same time it is a slow powder. In consequence it produces a pressure which increases more slowly, and is of longer duration. It does not break the material *in situ* into small fragments, a quality which is very essential for certain purposes, for instance, in the mining of coal, where it is desirable to break the material into pieces as large as possible, although the substance is quite brittle and easily cracked. Black powder will break an empty projectile into a smaller number of pieces with less effort, and which can then be driven further for the same expenditure of energy, less of it having been consumed in the work of crushing.

On the other hand, black powder has less effect, and does not break rock in a mine according to the direction where the mass is very compact and strongly adherent. It is easily turned aside, especially if the tamping or charge has not a greater resistance than that of the direction in which the rock has the least resistance. On this account it is necessary to make the drilling holes very deep and sometimes inclined at an angle of 45° for the purpose of giving a convenient length to the tamping, which is an element of expense.

The masses which become detached in the direction of the least resistance are frequently thrown to some little distance by black powder.

The cracks and cleavages adjoining the charge lessen the effect of the explosion. They weaken it until the effect is nullified; if these cross the drilling the expansion of the gases from the powder frequently takes place in the interior cavities, it is then that the mine *blows.* Also, in fissured rocks much time is often lost in closing up with rammed clay the cracks which are connected with the drill hole, whereas this work is useless when dynamite is used. For similar reasons it is of little value in blasting argillaceous or aquiferous rocks and calcareous tufas, in conglomerates, in beds of sand of high resistance, in one direction, and slight resistance in others.

It has little effect, for directly opposite reasons, in very hard and tenacious rocks, such as quartzite and certain felspars.

These circumstances, in addition to the fact that black powder has less force (the effect of one part of dynamite is regarded in practice as equivalent to two and one-half parts of black powder), explain the preference given to dynamite in most mining operations.

Nevertheless, black powder possesses certain advantages due to the gradual increase of the pressure which allows it to transmit its effects to a distance, for instance, in beds of coal, or better still, in wood, following the direction of the fibers. In recent earthworks, the pressures, if too suddenly produced by the quick powders, will shatter the mass and expend themselves in local work without much effect, while the slower tension of black powder displaces the earth and throws it in the direction of the least resistance.

From these details and these examples, we see what part the rapidity of the explosion plays in the transformation of energy into work.

9. Finally, the force of the explosion may be expressed in terms of the pressures produced, and by the work which they perform. The pressure is produced from the volume which the gases occupy at the temperature of the explosion. The work is due to the heat produced, and to the rapidity with which the gases are developed. These fundamental conditions —volume of the gases and the heat—are the consequences of the chemical decomposition; any reactions which liberate gases, or which augment the volume of a previously existing gas, may be the cause of an explosion. Therefore, in consideration of the foregoing observation, to define the force of an explosive, the following data are necessary:

First. *The chemical composition of the explosive.*

Second. *The composition of the products of explosion.*

The latter may vary during the different periods of temperature which succeed each other from the first moment of

ignition; it is also necessary to take the *dislocation* into account.

These three elements, the chemical composition of the exploding substance, the chemical composition of the products of the explosion, and finally the dissociation will be studied under the general title of *chemical composition.*

Also, it is desirable to define the following:

1. The *quantity of heat* given off during the reaction.

2. The volume of the gases formed under normal pressure.

These propositions are also obtainable from a knowledge of the two first in all reactions, which are positively known.

3. The rapidity with which the reaction takes place gives rise to the following studies, which are essential in order to be able to furnish a complete definition of explosives.

Origin of the reaction—The rapidity of the increase of the reactions.

To this sequence of ideas there should be attached a collection of phenomena, which are designated by the name of *explosions by influence.*

These phenomena, which have only been known for a few years, have seemed to us of sufficient importance to be separately considered with their accompanying developments.

In the treatment of these questions, we shall endeavor to cover all of the general ideas known at present concerning explosive substances.

II.

The Duration of the Explosive Reactions.

§ 1.—*Origin of the Reactions.*

We shall now take up the consideration of the chemical transformation of explosives, from the point of view of their origin and rapidity.

1. We will at first treat of its origin, that is to say, of the conditions which determine the commencement of the reaction. This, once started, maintains itself, and increases either by a simple progressive burning or by an almost instantaneous detonation.

Thus far, artillerists have expressed this origin by the expression "*set on fire,*" which implies that fire is applied locally to begin with; but the study of explosives shows us that the origin of the reaction may arise equally as well from a shock, from pressure, from friction, or from some other analogous mechanical force.

At first, suppose that it is necessary to refer all explosive reactions to an original heating, which is increased step by step, by successively bringing all the particles of the substance up to the temperature of its decomposition. The shock, the pressure, the mechanical conditions are not efficacious except as they determine this first heating, according to mechanisms otherwise different, and to which we shall return in the following paragraphs.

2. This being understood, the decomposition of the same material can take place at widely varying temperatures, but with equal rapidity, a material slowly decomposed at a given temperature being able to resist at much higher temperatures, though for a time continually decreasing as the temperature rises. Elsewhere I have explained all of this theory, (*Essai de Mecanique Chimique*, vol. II., p. 58, et seq.), and it is recalled only for the purpose of thoroughly fixing the ideas which are developed there. It plays a very important rôle in the explanation of the mode of formation of the secondary compounds produced in the explosion of powder; several of these compounds are formed all of a sudden, at a temperature which destroys them slowly, if they were maintained at that heat during a sufficient length of time; but the abruptness of cooling preserves the compounds, such as formene, ammonia, nitric acid from the destruction toward which they would hasten, because it brings them to temperatures at which they are perfectly stable.

3. This is the place at which it is desirable to introduce some expressions on the *sensibility of explosives.* This sensibility is equally dependent on the conditions of heating and of the method of propagating the reactions. It varies according to the conditions. Some substances are sensitive to the slightest elevation of temperatures, others to a shock, properly so-called, others detonate at the least friction. Silver oxalate detonates near 130°, nitrogen sulphide about 270°, mercury fulminate at about the same figure, somewhere near 90°, nevertheless,

the fulminate is much more sensitive to a shock and friction than the nitrogen sulphide and the silver oxalate. Thus we discern the special properties depending upon the individual structure of each substance, particularly for solids. But there also exist general conditions which it will be useful to define here.

4. The sensibility is greater for the same substance when operated on at a higher initial temperature, that is to say, at a temperature nearer to that which the substance begins to spontaneously decompose.

A fortiori the sensitiveness will be still further increased, if this limit is exceeded, that is to say if conditions occur where a slow decomposition may be transformed by the slightest heating into a rapid decomposition. A substance within these limits may be said to be in a state of *chemical tension*, an expression which is sometimes erroneously employed with reference to stable bodies, or for mixtures which have no habitual tendency to enter into a spontaneous reaction.

We have an example of such a case in *celluloid*, a body which does not detonate when struck by a hammer at ordinary temperatures, but it acquires the property of detonating when it is heated up to the point where it becomes soft, that is to say, up to about 160° to 180°, a locality which is near the temperature at which the substance decomposes.

5. When two different explosives are compared, which are decomposed at the same temperature, and with similar rapidity, their sensitiveness relative to shock and to friction at a lower temperature depends primarily on the quantity of substance on which the work of the shock expends itself; that is to say, it depends upon the cohesion of the substance which governs the transformation of the shock into heat. Cohesion, likewise, interferes with direct ignition, as the same quantity of heat produced by the combination of the first portions can elevate to the degree of decomposition, the temperature of a small quantity of water to which it is exclusively applied, which, if it is distributed over a larger mass, the temperature of that mass will not be brought up to the required degree.

6. The mass heated remaining the same, and the materials being different, the sensitiveness will depend upon the temperature of decomposition, which, for example, is lower for potassium chlorate than for the nitrate; the chlorate gunpowder is more sensitive than that made with nitrate.

7. The sensitiveness depends, furthermore, on the quantity of heat set free by the decomposition, that is to say, the sensitiveness will be greater, other things being equal, if the reaction gives off a greater amount of heat.

8. This same quantity of heat will produce different effects in acting on the same weight of substance according to its specific heat. For instance if potassium chlorate, whose specific heat is 0.209, be substituted for an equal weight of potassium nitrate, whose specific heat is 0.239, in the composition of an explosive mixture, a powder more sensitive than the nitrate powder would be produced. This condition acts in concert with the lower temperature of decomposition and with the absence of cohesion in chlorate powder, so as to render them particularly dangerous.

§ 2.—*Molecular Rapidity of the Reaction.*

1. The chemical transformation in a detonating mass is propagated with a certain rapidity, a knowledge of which is desirable for theory as well as in practice. In reality the rapidity with which the gases are given off depends on it, and in consequence the rapidity communicated to projectiles in guns, as well as the effects produced in mines at the expense of rocks to be thrown down or the obstacles which engineering desires to remove. For, the heat given off by a given reaction may be employed almost entirely to heat the gases and to increase the pressure, provided the reaction is very rapid, while if the reaction is made slower it is dissipated without effect by radiation or by conductivity.

A given quantity of an explosive may in this manner crush, *in situ*, such portions of rock as it comes in contact with, its energy being consumed without any result, from an industrial point of view, on account of its instantaneous decomposition. If the development of the gases is less rapid, but is still quite fast, an equal quantity of explosive may, on the other hand, dislocate the rock by de-

veloping extending fissures and sharply striking those portions of rock which are the nearest, a result which is sought for by miners. It may also produce elastic displacements and an undulatory movement of the soil without any local disturbance, if the pressures are developed sufficiently slow, so that the rocks shall have had time to be displaced *en masse*, in which case the explosive will be found to have produced scarcely any useful result.

This question of the quickness of reactions plays an important part in the studies relating to explosives, and therefore I am led at this point to collect the experiences and results which they produce.

2. The quickness of a reaction may be considered in two ways, if it is to act upon a homogeneous system, and especially a gaseous system, surrounded by conditions of pressure and temperature identical in all its parts; also, if the system is submitted at one point to an elevation of temperature or to a shock capable of determining an explosion, which is then propagated step by step.

It is desirable to begin with the examination of the first case, which serves as a basis for all the theory.

3. Having, therefore, a certain body, or a certain mixture, capable of undergoing a chemical transformation when the entire mass is placed under the conditions of temperature, of pressure, or of vibratory motions, etc., it appears as if the reactions should be instantaneously developed in all parts at once. The sudden explosion of nitrogen chloride and nitro-glycerine seem at first sight favorable to this conception. Nevertheless, a closer observation proves that the molecular reactions as a general thing consume a certain amount of time for their accomplishment, even when they are giving off heat.

Such, for example, is the decomposition of formic acid into hydrogen and carbon dioxide, which furnishes experiments that are easy to follow on account of the slowness with which their decomposition takes place. Operating in a closed vessel, and kept at a fixed temperature of 260°, it requires quite a length of time. And still this reaction gives 5.8 calories to each equivalent of formic acid, that is to say, 126 calories to the grain.*

4. The following are other examples of reactions which give off a great quantity of heat without being instantaneous. Thus acetylene, changed into benzine at a dark red heat by a slow reaction, gives off, without increase of volume, one and a-half times as much heat as a detonating mixture composed of oxygen and hydrogen in the proportions which form water, that is, 85.5 calories for 33.6 liters of acetylene (reduced at 0° and to 0m. .760) instead of 59 calories produced by the formation of vapor of water, by means of the same volume of detonating gas. It is about four times the amount of heat given off by chlorate powder for the same weight, that is 2,192 calories for each grain of acetylene transformed, instead of 590.6 calories for each grain of potassium chlorate powder.

The cyanogen gives off three times as much heat (1,435 calories to the grain) as the same weight of chlorate powder; or again, twice the amount of heat disengaged by its own volume of a detonating mixture formed of oxyhydric gas, such as 33.6 liters; 112 calories instead 59, when the so-called cyanogen is decomposed into carbon and nitrogen by the electric spark. Although the carbon begins to be precipitated almost immediately, still the cyanogen does not detonate in consequence of the spark, a fact which demonstrates the slowness of the reaction thus determined. Under other conditions, however, the cyanogen and the acetylene may be decomposed into their elements accompanied by detonation, but it is not by simply heating nor by the action of the electric spark.

I might go on multiplying such facts† which refer to the explosive bodies properly so-called when they are kept at a temperature slightly lower than that which determines the explosion. Silver oxalate, for instance, slowly decomposes at 100°, whereas, at a temperature a little above this it detonates strongly.

5. In brief, all molecular reaction, operated by simple heating at a constant temperature, in the midst of a homogeneous body, are surrounded by conditions which appear identical for all its parts, is

* *Essai de Mécanique Chimique*, Vol. II., p. 17.
† *Annales de Chimie et Phisique*, 4th series, 18, 142.

effected by a characteristic coefficient depending on the length of the reaction. This coefficient varies with the temperature, the pressure, the relative proportions; it plays an important rôle in the study of the powers of destruction among explosive compounds.

6. Let us follow out this explanation. The greater or less duration of a reaction does not change the quantity of heat given off by the total transformation of a given weight of explosive material. But if the gases which are formed expand in volume in consequence of the change of capacity caused by the escape of the projectile, or else by the cooling due to the contact with the walls; under such circumstances I say the initial pressures will be proportionally less than when the transformation of a given weight of an explosive will be of longer duration.

On the other hand, when a very rapid transformation of the entire mass in the midst of a closed vessel, added to the absence of the phenomena of dissociation, permits the initial pressures to reach the extent of their theoretical limits, or to approach them, it would be extremely difficult to make vessels strong enough to retain the gases of explosion.

7. The same state of affairs prevails, not only for an explosive body placed in a fixed and resisting volume, but also for the same body placed in a thin envelope, or beneath a layer of water, or even in the open air. In reality when the length of the reactions decrease beyond measure the gases given off develop pressures which increase with immense rapidity, so rapidly, indeed, that the enveloping bodies—solids, liquids, or gases—have not sufficient time to move and yield gradually to the pressure; these bodies oppose the pressure of the gas with a resistance comparable to that of a fixed wall. It is known that a pellicle of water on the surface of nitrogen chloride is sufficient to produce such results. The more instantaneous the reaction is the more the initial pressure, even in an open vessel, approaches the theoretical pressure, the latter being calculated for a case of decomposition under a constant volume, entirely filled by the explosive substance. It is in this way that we can explain the extraordinary effects of destruction produced by mercury fulminate, nitro glycerine, or compressed gun-cotton.

8. *As a general thing, any reaction that gives off heat is capable of producing explosive phenomena, provided, however, that it produces gaseous products,* and this for several reasons—First: *The rapidity of the reactions in a homogeneous system,* other things being equal, *increases with the temperature.** It even increases according to a very rapid law, as has already been shown by my experiments on the ethers; † hence the rapidity may be represented by an exponential function of the temperature, a function whose numerical value in the formation of acetic acid is 22,000 greater at 200° than when it is in the neighborhood of 7°. Secondly: *The temperature of the system increases, at least up to a certain limit, in consideration of the effect produced by the reaction.*

Let there be a system capable of giving off heat in consequence of its chemical transformation; if this system is confined in a locality where it can neither give up nor receive the slightest quantity of heat, the temperature of the system will continue to rise without stopping until it reaches a limit defined by a figure which is obtained by dividing the amount of heat given off, by the specific heat of the system. In addition, the rapidity with which this system tends towards this limit will increase in proportion to the extent of the elevation of the temperature already produced by the reaction, is greater.

In a gaseous system confined in a fixed space, the acceleration will become greater still, at least in the beginning, and that in consequence of the influence produced by the pressure, which pressure increases necessarily on account of the elevation of the temperature. For I have established the fact, that, all else being equal, and in operating at a fixed temperature the reactions take place more rapidly in liquid mixtures that in gaseous mixtures; it is especially noticeable that in gaseous mixtures the reactions are the more rapid according as the pressure is greater.‡ In a word:

* *Essai de Méchanique Chimique*, Vol. II., p. 64.
† *Ibid*, p. 93.
‡ *Essai de Méchanique Chimique*, Vol. II., p. 94.

Third. The rapidity of the reaction in a homogeneous system increases as the condensation of the substance progresses, or more simply *with the pressure in the gaseous systems.**

Thus, in an enclosure supposed to be impermeable to heat, the elementary rapidity of the reaction continues to increase, for the double reason that the temperature is continually being elevated and that the pressure of the gas increases without stopping. Nevertheless, the influence of the pressure should be more sensitive at the beginning than at the end of the experiment; provided, however, that the part which is not in combination, continually diminishes until the moment arrives when the proper tension of this part, considered by itself, ceases to increase in consequence of the heating; from that time on, it tends to diminish until it becomes null.

9. *The rapidity of the reactions in a homogeneous system depends upon the relative proportions of the components.*—In operating at a *constant temperature* the combination is generally accelerated by the presence of one or the other of the components.

On the other hand, at a constant temperature, the reaction is retarded by the pressure of an inert substance which has the effect of diminishing the state of condensation existing in the substance.

At a variable temperature the reactions are retarded *a fortiori* by the presence of an inert body, such as for instance the nitrogen of the air, or the silica of ordinary dynamite, this body by absorbing the heat lowers the temperature of the system without producing any special influence to hasten it by its pressure.

The reaction is generally slower at a variable temperature in the presence of an excess of one of the components, than if the operation is effected with equal equivalents, the necessity of heating this excess is more than counterbalanced by its accelerating influence.

It is clear that if the proportion of inert substance is such that the temperature of the system cannot be elevated to a degree necessary for the combination to continue of itself, the reaction will cease to be explosive and perhaps will not be propagated.

By this means the character of an explosive body may be changed—simply mixing it with an inert body. We shall now give some important facts. A seventy-five per cent. dynamite is not as sharp as pure nitro-glycerine, nevertheless, such a dynamite cannot be used for charging shells, for they would explode in the mouth of the cannon by the influence of the initial shock of powder.

Fifty or sixty per cent. dynamite, on the other hand, may be used with empty projectiles and can be fixed without giving rise to any injury to the ordnance.

This is not all, in using sixty per cent. dynamite, the projectile may produce an expression at the point it reaches without requiring any special priming, as for instance when its progress is stopped by a body of considerable resistance, as for instance a plate of iron sheeting, the elevation of temperature caused by this sudden arrest is sufficient to determine an explosion. But if the charge of nitro-glycerine be reduced to thirty or forty per cent. charged with such a dynamite, necessitates the use of a percussion force in order to produce an explosion similar to such as are produced by black gunpowder. It is quite true that such a dynamite presents scarcely any advantages over ordinary gunpowder.

It is an important observation that the rapidity of the burning of an explosive substance diminishes considerably as the proportions of its mixing with an inert body approach the limits of inflammability. It follows then, that as these limits are approached the burning becomes uncertain and the explosive character of the phenomena ceases to be manifested.

10. These general relations were established for such a system that all the heat which it gives off should be used to increase the elevation of the temperature. Let us examine the real condition of affairs, one in which the system gives a portion of its heat to the surrounding bodies by either radiation or conductibility. The elementary rapidity of the reactions, and the mass of the substances used, play an important part in this connection. In reality, on all occasions, when the rapidity of the reactions was

* In liquid or solid systems, the process, on the other hand, exercises but little influence, that is according to my investigations. A circumstance which is explicable because it is produced in consequence of the state of condensation of the material.

not great, a portion of the heat would to a certain extent be dissipated, and the elevation of its temperature soon reaches a fixed limit.

This limit will be one at which the loss of heat produced by the external action is equal to the gain resulting from the internal reactions of the system; in this case the reaction takes place with certain rapidity, constant or almost so, without becoming explosive. Such is the case with priming substances.

This is also the case, in explosives generally slower, of a substauce which is spontaneously decomposed. But if the mass with which the operation is conducted is increased, and supposing it to be confined in a fixed space, the amount of heat lost by radiation or conductivity at a given temperature will vary but little, the entire amount of heat produced internally will be increased.

Thus, the temperature of the system should be higher than the preceding when it tends toward a new limit, or when its growth becomes more and more rapid, and finally explosive in consesequence of the correlative growth of the pressures.

This same acceleration. depending on the pressures and the rapidity of the reactions, plays an important role in the interpretation of the effects produced by tamping.

Besides, it is in this means that all deflagrating mixtures may be changed into explosive compounds when the mass continued in a given space is increased.

The difference between the methods of decomposition of an explosive material, according as its mass is greater or less, deserves especial attention, for it is frequently referred to in practical applications.

11. This is observed, even in the case where an exit is opened to the gases of explosion. If the explosive mass is of sufficient size, the decomposition of a deflagrating substance where gases are given off through a narrow opening, may be changed into an explosion when the opening is made narrower, in such a way that the pressure and the internal temperature may be increased towards a given limit.

The same remark may be applied to spontaneous decompositions, occurring with large masses of matter. Beginning slowly at ordinary temperatures, their rapidity increases under the influence of the temperature which they determine; besides it may happen that this will change the character of the decomposition, by causing a new reaction giving off more heat to follow the initial reaction. The elevation of the temperature of the mass increases and hastens until it produces a violent reaction and a general explosion.

12. These facts which are frequently observed in laboratories, have been called upon to explain the spontaneous explosions of gun cotton and nitro-glycerine. They lead to the belief that an explosive substance which has begun to decompose is particularly dangerous. Such general explosions are produced not only in explosives that are contained in very solid vessels, but also in those which are held in vessels that are slightly resisting, such as boxes of wood or their thin metallic cases and even on substances heaped up in the open air, when the accumulation of substances allows the temperature to be raised of itself and to become more and more accelerated.

They may take place equally as well on substances divided into very small quantities, provided that the particles are sufficiently near to each other so that the mechanical effects may be accumulated and produce a common result.

In their preservation and in use the same precautions should be followed, just as if all the portions of the explosive were collected in a single mass. These are consequences which are theoretically possible and which are often proven to be practically correct by the accidental realization of terrible catastrophes.

13. In fact, the experiments made by the Birmingham Chamber of Commerce relative to the transportation and storage of caps, showed that the capsules, each containing 15 mgrs. of fulminate, will not explode in mass, nor by the influence of a shock, nor when crushed by the wheel of a locomotive, nor when they are placed in the center of an incandescent muffle, or when in the midst of a burning hearth.

But if the weight of the fulminate contained in the capsule is considerably increased, the case becomes different. The security which the first trials excited has created even in England, in conse-

quence of an explosion on the Thames of a boat which was loaded with detonating caps.

Experience has shown that beyond a doubt that the explosion of a single powerful capsule of fulminate is sufficient to cause that of all the capsules placed in the same box; if the box itself explodes the neighboring boxes will detonate equally as well.

It is in consequence of similar phenomena that the small fulminating caps which are sold as playthings to children, have so frequently been the cause of serious accidents.

At Vannes, near Paris, a child was amusing itself by exploding such a cap between the blades of a pair of scissors, two packages of 600 caps each, which were laying on the table close at hand went off at the same moment; the child was killed, the chair destroyed, and the floor injured.

We also add the explosion that occurred in the Rue Beranger, at Paris, on May 14, 1878, which was produced by a mass of fulminating caps that were intended for children's toys. These caps had the following composition: one kind called single consisted of—

Potassium chlorate	12 parts.
Amorphous phosphorus	6 "
Lead oxide	12 "
Resin	1 "

and those called double were made of a mixture of—

Potassium chlorate	9 parts.
Amorphous phosphorus	6 "
Antimony sulphide	1 "
Sulphur sublimed	0.25 "
Niter	0.25 "

The latter were more sensitive to friction, and on an average weighed 10 mgrs. each. Six to eight million caps of this description done up in rows of five and pasted on strips of paper, were piled up in the store in boxes containing a gross in each. Some one of these individual caps was set off by an accident, whose origin was never known, and a general explosion ensued. Of a sudden the house was thrown down, its facade destroyed by hurling the trimmed stones out of their positions. A stone, a meter cube in size, was thrown to a distance of 52 meters, a large portion of the adjoining house was also ruined; fourteen persons were killed on the spot and sixteen wounded.

These terrible effects are explained when it is recollected that the explosive material contained in the cap weighed about 64 kilogrms., and according to the composition of the substance that its force was equal to 226 kilogrms. of black powder.*

It is, therefore, of the greatest importance that persons having explosives in their charge should be familiar with these truths and facts, and adopt such precautions as will prevent the explosion of the entire mass.

III.

§ 3.—*Rapidity of the Propagation of the Reaction.*

1. Let us now examine the case of a homogeneous system, but whose various parts are exposed to different conditions, being such as those which arise from ignited at one point or from a local shock. In order to propogate the transformation in a mass which detonates, and which is not submitted to the same action at all of its points, it is necessary that the same physical conditions of temperature, of pressure, etc., which prevail at one point of action, should successively be produced and propogated, molecule by molecule, through all portions of the mass.

In this connection the numerous works of artillerists are well known† on the rapidity of combustion of ordinary gunpowder, and of that of gun-cotton, a capacity which varies according to the physical structure of the powders and their chemical composition. We shall presently examine these results as well as those observed in explosive mixtures of gases, that is to say the observations bearing on the rapidity of the combustion of mixtures of oxygen and hydrogen, or of nitrogen oxide or gaseous hydrocarbons.

Then we shall give some new and unexpected results furnished from the study of gun-cotton and of nitro-glycerine, the new theory of the functions of caps, the distinction thus far ignored between the

* These facts are taken from the report made the Inquest Commission.

† Piobert, Praste de artillerie, partie theorique.

simple ignition and the true detonation of explosives, a distinction which my recent investigations extend to even gaseous mixtures themselves, and we shall seek to harmonize their differences with theoretical ideas.

2. According to Piobert, the rapidity of the combustion of powder in the open air observed on prisms of known length, placed vertically, and whose lateral faces were greased in order to insure regularity in the phenomena. This rapidity, I say, has been found to be included between 10 and 13 mm. to the second in gunpowder. Otherwise it varies in inverse proportion to the apparent density of the powder.

3. The rapidity of combustion of powder depends to a great extent on the pressure of the air or on the surrounding gases.

Near the end of the seventeenth century Doyle made some experiments on the combustion of powder in vacuo, and observed that grains of powder thrown on a red-hot iron in this condition, melt without detonating. If the operation is conducted with a sufficient number of grains, towards the end an explosion will take place beyond a doubt, because the conditions of pressure are changed.

Huygens repeated the same experiments by igniting the powder with a burning-glass which concentrated the solar rays.

If the heating is progressive, an effect which may be produced by a piece of glowing charcoal then at pressure, the sulphur will be sublimed and the homogeneity of the mixture destroyed, or else according to Hawksbee (1702) the powder will be melted.

These experiments have been frequently repeated with different modifications, such as the employment of a red-hot platinum wire, heated by electricity, and then used to ignite the powder in vacuo.—*Abel.* M. Bianchi has in this manner determined that gun-cotton is slowly decomposed in vacuo before its explosion, and a similar result with nitroglycerine has been reached by Messrs. Heeren and Abel.

Mercury fulminate, on the other hand, detonates in vacuo when brought in contact with a piece of brass wire which has been heated red-hot, but the detonation does not extend to the grains which are not contiguous to it, as it does when under atmospheric pressure.

4. Not only does a vacuum reduce the explosive qualities of gunpowder, but any diminution in the pressure retards it. In 1855, Mitchell observed that fuses burned slower at high elevations; M. Frankland in 1861, at his laboratory, and then M. de Saint Robert on the Alps, have made very exact determinations in this line, under the pressures included between 722 m.m. and 405 m.m., according to the researches of M. de St. Robert, rapidity of combustion of the powder under less than atmospheric pressure would be represented for all practical purposes by a formule such as $V = Ap^{\frac{2}{3}}$.

A being a constant and p expressing the pressure. These results should be attributed to the greater or less rapidity with which the heated gases escape before having had time to heat the neighboring portions of the solid matter, which is equivalent to saying that the pressure diminishes the number of gaseous particles carried up to a high temperature, that come in contact at each instant with the solid particles not yet ignited, and share with them their living force in a way so as to produce an equilibrium of temperature.

Whatever may be the pressure, the initial temperatures of these particles is pretty much the same at constant volume, at least so much as has not been modified by the chemical reaction. But if one operates under a constant pressure, it is otherwise, for the temperature is lowered in accordance with the detention of the gases.

5. On the other hand, the quickness of combustion of powder increases with great rapidity, as soon as it attains the heavy pressures which are produced in cannons and in guns; thus, for instance, Captain Castan reckons the rapidity of the combustion of powder in the bore of cannons of large calibre at 230 mm. per second, instead of 10 mm. in the open air.

6. The rapidity of combustion of other explosives has not been the subject of experiments as exact as those applied to black powder; it likewise suggests new observations and a theory of an entirely different kind, as we shall immediately explain. We confine ourselves to the statements that Piobert determined the rapidity of combustion of gun-cotton (not compressed) as eight times that of

gunpowder, a value which may be applied to a progressive combustion taking place without detonation.

7. These same studies were extended to explosive mixtures of gases. In 1867, M. Bunsen * determined the rapidity of combustion to be 34 m. to the second for detonating gas (hydrogen and oxygen) and of one meter only per second for a mixture of equivalent parts of carbon monoxide and oxygen. These mixtures being taken at atmospheric pressures. He determined the delivery through a small orifice by igniting the jet, and determined for what rapidity as a limit of flow the flame remained stationery at the opening without going back into the interior. M. Mallard † has made similar experiments in different mixtures of marsh gas or of illuminating gas and of air; he found that the rapidity of combustion, defined as above, rapidly diminishes in proportion as the amount of gas having no part in the combustion increases. The maximum rapidity corresponding to 0.560 m. each second for a mixture of eight parts of air and one part of marsh gas by volume. It lowers itself to 0.04 m. with a mixture containing twelve parts of air to one of marsh gas. With illuminating gas and air, the maximum rapidity has almost reached double this amount mm. Mallard and Le Chatelier have restudied this question by other processes which have given them results entirely different according to the method of combustion. They will be referred to presently and the causes of these differences will be shown.

8. In reality the study of the new explosives, gun-cotton and nitro-glycerine, lead to a better understanding in the knowledge of the means of propagation of the chemical reaction in the midst of a mass during combustion, and it has greatly modified the ideas which have been held on this subject. Formerly, when black powder was the only known explosive, its ignition was all that demanded attention, the effects of the explosion that followed did not appear depependent on the process of ignition. But nitro-glycerine and dynamite have shown singulur differences in this connection.

9. In order that these may be perfectly understood, it is necessary to first consider those phenomena of shock and other analogous causes capable of producing a deflagration.

The shock will hardly produce by itself the decomposition if a substance absorbs heat, unless we refer to colossal masses animated by enormous living force, and which are concentrating all their action on a very small quantity of matter, something which is very difficult to produce. For instance, the living force of a weight of 1630 kilograms falling from the height of a meter, would be necessary to decompose one grain of water, that is by supposing that it would be possible to transmit to a grain of water, by any means, the entire amount of this living force.

On the other hand, if the decomposition of the substance gives off heat, one would suppose that a living force which was limited would be sufficient to produce it, provided that it was in condition to be applied completely to a very small quantity of substance which it raises to the degree of temperature necessary in order to determine the reaction.

Thus, for instance, several strokes of a hammer violently struck on some powdered potassium chlorate, wrapped up in a sheet of platinum and placed on an anvil, is sufficient to produce traces of potassium chloride that are quite perceptible; while potassium sulphate will give no indications of decomposition under the same conditions. But it must be remembered that the decomposition of potassium sulphate into potassium sulphide and oxygen absorbs heat, while the decomposition of potssium chlorate into potassium chloride gives off heat, (11,000 calaries for potassium chlorate).

10. This condition is, however, not sufficient to cause a shock to produce detonation. It is still necessary that the live force, developed by the decomposition of the first portions, should be communicated to adjoining portions, in such a way as to determine, step by step, the decomposition of the entire mass. The shock from the hammer which is not sufficient to produce these conditions with pure potassium chlorate is, on the other hand, efficacious with nitro-glycerine. The fall of a weight 4.7 kgrams. from a height of 0.25 meter is sufficient to cause the explosion of a single drop of nitro-

* *Annales de Physique et de Chimie*, 4 Serie t. 14, p 449.
† *Annales des Mines*, t. 8, 3c. Herason, 1871.

glycerine occupying a surface of 2 c.cm. square.*

Nitro-glycerine mixed with infusorial earth constitutes dynamite, a substance which is but slightly sensitive to shock because of the porous and cellular structure of the silica prevents to the immediate and local communication of the living force of a very slight quantity of nitro-glycerine separated from the rest.

Besides, the explosion of black powder will cause the nitro-glycerine to detonate, but it will not lead to any explosion of dynamite, at least in the open air and in weak charges. But this inertia disappears under the influence of certain shocks, particularly violent, such as that of mercury fulminate. The explosion of nitro-glycerine varies according as it is pure or mixed with some other body, whether it is effected by a simple shock, by the contact of a body in feeble ignition, or in strong ignition, or an ordinary match, or else by the contact of a strong mercury fulminate cap.

IV.

1. According to the process used for igniting, the dynamite may be decomposed quietly and without flame, or else it may burn with considerble vivacity, or else produce a detonation, properly so called, which may be sometimes moderate, sometimes capable of dislcoating rocks, sometimes even of destroying them *in situ*, and producing the most violent effects.

2. The substances, which are the cause of these last-named results, have been specially designated as *detonators*. M. Nobel was the first to observe these effects on nitro-glycerine (in 1864), and he then deduced a suitable process by which it could be made to detonate with certainty by means of a cap containing mercury fulminate. Gun-cotton does not show any less difference. M. Abel has published in this connection since 1868, many very curious experiments, and which tend to establish a great diversity between the conditions of deflagration of this substance varying with the method of ignition.† M. Roux and Sarran have generalized these phenomena by distinguishfng what are known as *explosions of the first and of the second order*.

3. However strange this diversity may appear at first sight, I nevertheless believe that the thermo-dynamic theories are capable of accounting for it, by a suitable analysis of the phenomena of the shock.

In truth, the diversity of the explosive phenomena depends upon the rapidity with which this reaction propagates itself, and the more or less intense pressure which results from it.

Let the case be a more simple one, such as an explosion caused by the fall of a weight from a certain height. At first one would be disposed to charge the effects observed to the heat given off by the pressure due to the shock of the weight suddenly arrested. But calculation shows that the arresting of a weight of several kilogrammes, falling 0 25 m., or 0.50 in height, would not be capable of raising the temperature of the explosive mass more than a fraction of a degree, if the resulting heat was dispersed uniformly throughout the entire mass; this would not then attain an elevated temperature, that of 190 degrees, for instance, for nitro-glycerine, a temperature to which it appears necessary to suddenly raise the entire mass in order to produce an explosion.

It is by another mechanism that the living force of the weight which is transformed into heat becomes the origin of the observed effects.

It is sufficient to admit that the pressures which arise from the shock exercised on the surface of the nitro-glycerine being too rapid to become uniformly dispersed throughout the entire mass, the transformation of the live force into heat takes place especially among the first layers reached by the shock. If it is sufficiently violent it may thus be rapidly elevated to 200°, and they will be immediately decomposed and producing a large quantity of gas. The production of gas is in its turn so violent, that the shocking body has not had time to displace itself, and that the sudden detention of the gases of explosion produces a new shock probably more violent than the first on the layer situated below. The living force of this same shock is changed into heat in the layers

* Ch. Girard, Millot et Vogt, *Comptes Rendus de Seances de l'Academie des Sciences*, tome 71, p. 391

† *Comptes Rendus des Seances de l'Academie des Sciences*, tome 69, p. 105-121. 1869.

which it first reaches. It produces the explosion and this alternative between a shock, developing a live force which changes into heat and a production of heat which elevates the temperature of the heated layers up to the degree of a new explosion capable of reproducing a shock; this alternative, I say, propagates the reaction, molecule by molecule, through the entire mass. The propagation of the deflagrating takes place this way in consequence of phenomena comparable to those which produce a sonorous mass, that is to say, by producing a real explosive which advances with a rapidity incomparably greater than that of a simple burning provoked by the contact of a body in ignition and operating under conditions where the gases expand freely in proportion to their production.

4. This is not all. The reaction started by the first shock in a given explosive material is propagated with a rapidity which depends upon the intensity of the first shock, provided that its living force, changed into heat, determines the intensity of the first explosion, and in consequence that of the entire series of consecutive effects. For the intensity of the first shock may vary considerably, according to the method by which it is produced. The effect of a blow from a hammer may vary in its duration—for example—from the one-one-hundredth to the one-ten-thousandth of a second, according as one strikes with a hammer having a flexible handle, or with a block of steel, that is according to the experiments of M. Marcel Duprez. From this it may be seen that the explosion of a solid mass or a liquid may develop itself according to an infinite number of different laws, each one of which is determined, all other things being equal by the original impulse. The more violent the initial shock, the greater will be the resulting violence of the decomposition and the greater will be the pressures which are exercised during the entire converse of this decomposition. One and the same explosive substance may produce very different effects according to the method of ignition.

5. The effects likewise differ as the substance is pure or mixed with a foreign substance, and in accordance with the structure of the latter. This feature is shown by dynamite, a mixture of nitro-glycerine with silicon, which has lost the greater part of its sensibility to an ordinary shock, but remains explosive to the shock of a ball, and above all, to that of mercury fulminate.

The addition of a few per cent. of camphor to dynamite will still further diminish its explosive qualities to such an extent even, that it will no longer detonate except with strong fulminate caps.

6. Gun-cotton, impregnated with water, or with parafine, becomes likewise insensible to a shock; for its detonation, then it requires the use of a small supplementary cartridge of dry gun-cotton, itself charged with fulminate.

If several per cent. of camphor are mixed with nitrated cellulose, its susceptibility to explode by a shock is almost completely destroyed, at least at ordinary temperatures; to such are that this mixture forms a substance which is used to-day for many purposes in the arts under the name of *celluloid*.

7. The dynamite given, which results from the combination of nitro-glycerine with collodion, sometimes with camphor added, also forms an elastic mass which is only slightly sensitive to shock, and it also requires an auxilliary cartridge of dry gun-gotton, itself charged with fulminate.

8. The change brought about by the addition of camphor and resinous substances to the explosive power of similar substances, is the result of a modification brought about in the cohesion of the mass. This has acquired a certain elasticity and a solidity of parts, in consequence of which the initial shock of the detonator propagates itself at first in a much greater mass. Besides a portion of the effects are expended by the work of tearing up and separation, there still remains the smaller portion which is capable of producing heat in the parts directly struck, this heating, however, being dispersed through a larger mass.

Therefore, a sudden elevation of temperature at one spot, capable of producing consecutive chemical and mechanical action can only be produced with difficulty; it requires the employment of much greater weight of the detonator. This follows always in consideration of the preceding theory.

9. But camphor, on the contrary,

should not produce, and does not, as experience goes to prove, any specific action on a discontinuous powder, such as the potassium chlorate powders. It is on this account that it is necessary, equally as well to take into consideration that frozen dynamite jelly possesses a sensibility to shock comparable to that of nitro-glycerine if the solidity of the parts have become destroyed by the crystallization of that substance.

10. The importance of *caps* may be clearly seen from the foregoing. Up to the present time they have been regarded simply as agents, seeming to communicate the ignition to the powder. In reality, these caps, as long as they are of sufficient size regulate by their nature the character of the initial shock, and in consequence the character of the entire explosion. In this case they receive the name of *detonators*, properly so called. Mercury fulminate is used for this purpose particularly, on account of its being more powerful, that is to say, its shock is more violent and more sudden than that of any other substance, which is explained by the greatness of the pressure which it develops by detonating within its own volume (nearly 40,000 atmospheres).

We have given above a certain number of characteristics relative to the spinal influences of caps. We shall return to this subject.

V.

§ 4.—*Burning and Detonation.*

1. The term *burning* is specially given to progressive combustion, the expression *detonation* being reserved for rapid and almost instantaneous combustions. Hence we obtain the distinction proposed by M. Sarran between *detonations of the first order*, such as those of black powder, whose detonation is the starting point of the series of burnings, and the *detonations of the second order*, or detonations proper, such as those of nitro-glycerine induced by a powerful cap of mercury fulminate. However, the known facts do not, in my opinion, oblige us to admit of a difference of nature or of a sharp line of demarkation between the two varieties of phenomena. They tend rather to cause them to present an indefinite variety, included between the two extreme limits, as follows:

First. *The detonation of the explosive in its own volume*, reaching the maximum of temperature and of pressure, and in consequence, the maximum of rapidity of which the chemical reaction taking place under these conditions is susceptible. This detonation is specially induced by a very quick shock. The gases formed at the point where the shock is produced has not, so to speak, the time to be displaced, and so communicate their living force to the parts in contact; the action is thus propagated throughout the entire mass with a sort of regularity. It is to this order of detonation that the rapidity of propagation belongs, so different from that of the combustion of black powder, which has been measured in comparison with dynamite and compressed gun-cotton. For instance, Austrian artillerists have noticed a rapidity higher than 6,000 meters a second in causing the detonation of a cylinder of dynamite 67 meters long. Colonel Sébert has observed rapidities of 5,000 to 7,000 meters in gun-cotton, powdered and compressed in long tubes of lead. Further on it will be seen that I, myself, measured with M. Vielle rapidities of several thousands of meters per second in mixtures of detonating gases taken at the ordinary pressure and contained in tubes of iron, of lead, or even of rubber.

Second. *The progressive burning* propagating itself from particle to particle in conditions where the cooling due to external agents lowers the temperature to the lowest degree compatible with the continuation of the reaction.

It is to this order of burning that we refer the rapidity of combustion of the detonating gases to, as measured by Bunsen. In the case of solid or liquid explosions, the propagation of a simple burning is rendered more difficult than otherwise by the movements of the gases which expand to a considerable extent around the point which is set on fire; instead of acting in an equal volume or in one slightly different from that of the primitive body, their temperatures are thus reduced by distribution through a greater mass of matter. The temperature is frequently found to be dispersed by the gases without giving rise to a total combustion or even undergoing any change. This is particularly the case

with explosions which are not forced into an envelope to concentrate the action of the gases and give them a common resultant.

This is the case with the nitro-glycerine which is found unaltered almost *in situ* in progressive explosions; such is likewise the case with dynamite that is laid along the ground in a thin layer. Damp gun-cotton that is not inflammable in the cold, also furnishes a number of illustrations of this dispersion, resulting from the use of an insufficient detonation. In is, in consequence of this reaction of the gases, that it is deemed advisable to present the simple burning of dynamite in caatridges from preceding the action of the fulminate.

2. Between these two limits there is observed an entire series of intermediate stages of an unlimited number, as is shown by the different methods of burning of dynamite, and the influence of tamping which allows the transformation from burning into a real detonation, if the tamping is sufficiently resisting. Finally we may cite the inequality of the effects produced by the successive explosions of the charges of the same agent, which detonate by influence within limited distances beyond which the explosion will not propagate itself.

3. We must also refer to chemical phenomena. That of decomposition prevails when the explosive substance contains sufficient oxygen to experience a complete combustion as occurs for nitro-glycerine and nitro-dynamite; besides it is necessary that this total combustion shall have actually taken place, which does not necessarily happen, especially in slow burnings performed at a temperature as low as possible.

4. But it often happens that the oxygen is insufficient, or that the first reaction gives rise to a wasteful expenditure of this oxygen as is the case when nitro-glycerine burns slowly, with the production of nitrous vapors and of fixed or gaseous substances incompletely burned. Under these circumstances the possible decompositions are numerous; their number depends on the temperature, on the pressure, and on the rapidity of the heating. We have already remarked upon this in the case of ammonium nitrate; it may be observed in general inorganic substances, decomposed by heating. (*Essai de Mecan. Chimique*, t. II., p. 45.)

5. Among these decompositions those which develop the most heat are those which produce the most violent explosive effects, other conditions being equal. This fact is evident when the volume of gas (reduced at 0° and 0.76°) reaches in the same time its maximum value. But it is also verified in other cases, the dissociation giving rise always to a diminution of pressure, as I have shown elsewhere. On the other hand, this fact does not obtain as a general thing in reactions which are produced at the lowest possible temperature. If then, the explosive body receives in a given time a quantity of heat insufficient to carry the temperature up to a degree, which corresponds to the most violent reaction, it will experience a decomposition capable of giving off less heat, or even of absorbing heat; and it is capable of destroying itself completely by this decomposition without developing these explosive effects which are most energetic.

The opposite will take place if the body is quickly heated to the temperature corresponding to the most energetic reactions.

6. Finally the multiplicity of the possible reactions carries with it a series of intermediate effects, and this all the more, because, according to the mode of heating, it may happen that several decompositions will follow each other progressively. This succession of decompositions gives rise to effects which are very complicated, as has been noticed by M. Jungfleish, when the first decomposition, instead of producing a total elimination of the decomposed portions (changed into gaseous or volatile substances) produces a division of the primitive substance into two parts: the one gaseous, which passes away; the other solid or liquid, which remains exposed to the consecutive action of heating. The composition of this residue being no longer the same, as is the case, for instance, with nitro-glycerine which first loses a portion of its oxygen in the form of nitrous vapors, the effects of its consecutive destruction may become completely changed.

7. Such are the causes, some chemical, others mechanical, through which nitro-

glycerine and compressed gun-cotton produce all of these different effects, according as they are ignited by the aid of a body in feeble combustion, or by a flame, or by an ordinary cap, or else by means of a cap charged with mercury fulminate.

For example, M. M. Roux and Sarran have found that the charge necessary to break a shell, other things being equal, will vary in an inverse ratio of the following numbers; these numbers being referred to gunpowder, taken at unity—

	Detonation.	Burning.
Nitro-glycerine	10.0	4.8
Compressed gun-cotton	6 5	3.0
Picric acid	5.5	2.0
Potassium picrate	5.3	1.8

The weight of the breaking charge in the case of gunpowder itself, under the influence of nitro-glycerine primed with fulminate, is capable of being reduced in the ratio of 4.34 to 1.

This inequality in the force of different powders is partially attributed to the cooling which is effected by the walls during a slow reaction, and also in part to the changed chemical condition.

8. The diversity is less marked with the non-compressed gun-cotton, because the influence of the initial shock is exercised on a smaller quantity of matter, and above all. because the propagation of the successive reactions in the mass develops their initial pressures weaker, and a less direct transformation of living force into heat, is transmitted to the explosive body on account of the air which is interposed and in consequence the explosive wave can hardly be generated.

Compressed gun-cotton itself, is not so compact as nitro-glycerine, because of its structure; on this account the pressures due to shocks might be sensiby weakened by the existence of interstices. Gun-cotton also detonates with more difficulty than nitro-glycerine. The nitro-glycerine will detonate by the fall of a weight from an insignificant height, that is, provided a cap charged with gun-cotton or a mixture of fulminate and of potassium chlorate, etc., is used; while on the other hand, gun-cotton does not explode by the influence of nitro-glycerine, nor by the influence of a mixture of fulminate and of chlorate. It requires the more violent shock of pure mercury fulminate. Besides, this is less efficacious if it is used unenclosed than when confined in a thick envelope of copper or tin plate; it is less powerful in an envelope of paper or tin-foil than it is when wrapped in copper: it is still less powerful if the cap is not in direct contact with the gun-cotton. Finally, if it is placed in the tube of a feather its effect will be annulled. Nitro-glycerine does not detonate as well when exposed to the influence of a fulminate force, if it is ignited before the explosion of the fulminate, this preliminary ignition having the effect of producing a certain vaccuum between the two.

The absence of an immediate contact between the dynamite contaiued in the cartridges and the fulminate cap is objectionable for the same reason, the shock being weakened in part by the interposed air. The sensitiveness, the action of the fulminate which contains the liquid nitro-glycerine, than in that which contains frozen nitro-glycerine which may also be explained as due to the lack of homogeneity of the frozen dynamite in which the nitro-glycerine is partly separated from the porous silica in consequence of its solidification.

9. All these phenomena may be explained by the more or less important value of the initial pressures, and by their more or less rapid development; that is to say, by the conditions which regulate the living force transformed into heat within a given time, into the midst of the first layers of the explosive substance reached by the shock.

The quantity of living force thus transformed depends first upon the sharpness of the shock and also upon the amount of work which it may produce; these are the factors which vary with every explosive material. For instance, it is not always that the most sensitive caps are those which produce the most instantaneous explosion. M. Abel has observed that nitrogen chloride is not particularly adapted for igniting gun-cotton; nitrogen-oxide, so sensitive to the least friction, remains entirely powerless to explode gun-cotton. Now, nitrogen chloride is precisely one of these explosive bodies which we are now discussing, which develops less heat and in consequence less work for a given weight. We conclude, then, that it cannot be advantageously employed for caps or priming. As

to nitrogen-oxide, according to the analogies drawn from the iode substituted compounds (see *Annales de Chemie et de Physique*, 4 series, vol. 20, p. 449) its explosion should develope still less heat and work for the same weight than the nitrogen chloride. Its weakness is therefore easy to comprehend.

VI.

1. The phenomena which we have described has had reference to solid or liquid explosives, but the gaseous compounds and the detonating mixtures of gases give similar results, these throw a still greater light on the theory. In reality the chemical transformation of such a gaseous mixture may act with very different rapidities according to the mode of propagation of the decomposition or of the burning.

2. Let us begin with a compressed gas formed with the absorption of heat according to its elements, such as hypochlorous and ($Cl+O=ClO$ absorbs -7.6), acetylene ($C_4+H_2=C_4H_2$ absorbs -61.5) or cyanogen ($C_4+N_2=C_4N_2-74.5$.

Such a gas is decomposed in inverse proportion to the liberation of heat. This, for instance, is what we obtained with the hypochlorous gas, heated below 100° or traversed by an electric spark, or placed in contact with a body in ignition; the gas detonates at once, reproducing chlorine and oxygen. But it is not the same with acetylene nor with cyanogen. These gases neither detonate by the influence of heating nor by the influence of the electric spark, although they are decomposed by them, little by little, and without exploding. On the other hand, I have found that a quick shock of mercury fulminate will cause them to detonate rapidly with considerable flame, separation of their elements, carbon and hydrogen in the case of acetylene, and carbon and nitrogen in the case of cyanogen. The experiments have recently been described in the present work.*

3. The explosive mixtures formed by the union of oxygen with a combustible gas, may also burn with extremely varying rapidities, according to the method of the propogation of the chemical reaction.

We have cited the experiments of M. Bunsen who thought it possible to fix the rapidities of combustion of a mixture of hydrogen and oxygen gas, in equivalent proportions, at 34 meters per second, and that of a mixture of carbon monoxide and oxygen at only a meter. These are experiments based on the retrogradation of the flame back into the mixture, while flowing out into the atmosphere through a narrow opening. Having recently undertaken these experiments with M. Viollé under different conditions, we have observed rapidities incomparably greater. Our method of operation was as follows:

On each occasion we filled, under atmospheric pressure with an explosive mixture, an iron tube 5 meters long, having an internal diameter equal to 8 mm., capable of being sometimes kept open and sometimes closed at its extremities, or else a lead tube 40 meters in length, or sometimes an thick india-rubber tube 40 meters long. By means of a special arrangement we are able to register the passage of the explosive wave, first at the beginning of the tube, then further on, and finally at the end of the tube,* and also we were able to measure the time elapsing between these different passages. These experiments were made with the tubes which were sometimes opened at one extremity and sometimes closed; on some occasions they were placed horizontally, and on others they were arranged vertically, and under different pressures. They had proved to us that the detonation is propagated with a rapidity equal to thousands of meters per second, as well for the mixture of hydrogen and oxygen as for the mixture of carbon monoxide and oxygen.

4. The difference between these results and those which were obtained by M. Bunsen may be explained by the variance in conditions. The gases burned in the earlier experiments were cooled by contact with the air, and the explosive wave was not produced. The difference between the two kinds of combustion appeared analogous to that which exists between the simple burning of explosive substances, an operation in which the movement of the different particles takes place confusedly and in an independent manner, and their sudden detonation, provoked by a fulminating cap, an opera-

* See *Comptes Rendus*, 93, p. 240.

* *Comptes Rendus*, 93, p. 19.

tion in which the movements become coordinated. Hence the effects of temperature and of pressure attain their maximum, and are propagated with an incomparably greater rapidity. Some of these observations made with the fire-damp of mines seems to bear an analogous interpretation.

5. The characteristic feature of this order of phenomenon is the production of an explosive ware, that is to say, of a certain regular surface where the transformation is produced and which brings about a similar state of combination, of temperature, of pressure, etc. This surface, once produced, propagates itself layer by layer throughout the entire mass, in consequence of the transmission of successive shocks of the gaseous molecules carried to a vibratory condition more intense in consequence of the heat given off in their combination and transformed *in situ*, or more exactly with a feeble relative displacement. Analogous phenomena may be developed with solid and liquid explosives in confirmation with what has been said above.

6. These effects are comparable to those of a sound wave; but, however, with this important difference, that the explosive phenomena does not reproduce itself periodically, that is to say, it starts a single and characteristic wave, whereas the phenomena of sound is reproduced by a periodical succession of equal waves.

There is, moreover, this important difference that the living force of a system of molecules, whose collection forms the sound wave, remains sensibly constant during the propagation of the wave, and that it is slight, whilst the living force of the system of molecules which constitutes the explosive wave is enormous, and that it begins at once by growing and tends towards a maximum, which is determined by the very highest temperature that the system can reach due to the chemical transformation actually realizable. In fact this maximum is never reached, in consequence of the conditions of cooling, but it is more nearly approached as the reaction is more rapid, is carried on in a medium more condensed and on a greater mass

7. The propagation of successive shocks between the ultimate molecules of bodies leads us to push further the comparison of the mechanical effects and the thermal effects which are developed simultaneously. In reality the living force communicated to this order of molecules by the chemical combination is nothing else than the heat itself given off in the reaction and the pressure exercised on the molecules themselves and on the walls of the vessels, is the immediate phase of transformation according to present theories. We have reached, therefore, a point where the two orders of ideas tend to confound themselves.

8. It follows from these explanations that the rapidity of the propagation of the explosion becomes comparable to the rapidity of sound, which is also propagated in virtue of an undulatory movement, the rapidity of these two movements being of the same order as the rapidity of translations of the gaseous molecules.

9. It is possible to define this point of view by observing that the rapidity of the translation of the gaseous molecules is equal, according to the formulas of M. Clasius to 29.354 meters $\sqrt{\frac{T}{P}}$ per second. T in this instance represents the absolute temperature $(273+t)$, p the density of the gaseous mixture in terms of that of air. Let T=3000°, a temperature whose development* may be admitted in the gaseous mixtures which we are considering taken at the normal pressure. The actual rapidity of the translation of the gaseous molecules would be included between 1,300 meters and 1,600 per second, according as one operates on carbon dioxide or on a mixture of carbon monoxide and oxygen, or on a dissociated mixture containing these different compounds. It would be included between 2,000 and 2,500 meters per second for steam or its compounds. These figures may furnish the first term of the comparison, though we must not forget that explosive phenomena are more complex than a simple movement of translation, or even than the propagation of a sound wave.

VII.

Explosions by Difference.

1. Thus far we have regarded the development of the explosive reactions

* *Ann. de Chem. et de Phys.*, 5 Series 12, 309.

either from the point of view of their duration in an homogeneous system in which all the parts are maintained at the same temperature, or else from the standpoint of their propagation in a system equally homogeneous, to which fire is applied directly by means of a body in ignition or else by a violent shock. In these last years, however, the study of explosive substances has revealed the existence of another method of propagating the reactions in an explosive center, this propagation taking place at a distance and by the intermediation of the air or certain solid bodies which do not themselves participate in the chemical change.

We now wish to speak of what are called explosions by influence, whose existence was formerly suspected from certain known facts relative to the simultaneous explosion of several buildings separated by considerable space from each other, as in catastrophes occurring in powder mills.

Attention has been specially directed to this class of phenomena by the study of nitro-glycerine and gun-cotton.

2. We will begin by giving the most important characteristic facts. A dynamite cartridge made to detonate by means of a fulminate cap causes the adjoining cartridges to detonate, not only by contact and by direct shock, but even from a distance. In this way an indefinite number of cartridges, arranged in a regular course, may be made to detonate.

3. The distances to which the explosion may be propagated are relatively great. Thus, for instance, cartridges being contained in rigid metallic envelopes and placed on a resisting soil, the detonation produced by 100 grains of Vonges' dynamite (75 per cent. nitro-glycerine, 25 per cent. randanite, that is to say, very finely divided silica) communicates itself 0.3 meters of distance, according to the experiments of Captain Corille. D being equal to the distance in meters, and C the weight of the charge in kilograms, the experiments of this officer show that D=3.0C.

When the caps were laid on a rail, D was found to be equal to 7.0C.

On soft or ploughed up earth the distances, on the contrary, are less.

When a cartridge is suspended in air there is no detonation by influence, perhaps because the cartridge not being fixed can recoil freely, which diminishes the violence of the shock. Nevertheless, there are experiments which show that the air suffices for the transmission of the detonation by influence, although with greater difficulty and requiring a greater mass of the explosive.

With a dynamite less powerful in nitro-glycerine (55 per cent. of nitro-glycerine and 45 per cent. of the argillaceous ashes of boghead coal), contained in similar cartridges, and placed along the ground, the experiments of Captain Pamard have given the smallest distances: D=0.90c. If metallic envelopes having less resistance are used, the distance at which the explosion is propogated is likewise diminished. Dynamite simply spread along the ground ceases to propagate the explosion. The experiments performed in Austria have given similar results. They have shown that the explosion is communicated either in the free air with intervals of 4 cm., or else through pine boards 18 mm. thick. In a lead tube with a diameter =0.15 meters and a meter in length, a cartridge that is placed at one extremity has caused the detonation of a cartridge at the other end. The explosion is still better transmitted through tubes made with wrought iron. Couplings of the tube diminish their aptitude for transmission.

4. An explosion which is propagated in this manner will go on weakening itself from cartridge to cartridge, and even change its character. Thus, according to the experiments made by Captain Müntz at Versailles, in 1872, a first charge of dynamite, exploded directly, excavated a funnel-shaped hole in the ground with a radius of 0.30 meters; the second charge, detonated by influence, produced an opening of only 0.22 meters; the effect of the detonation was then reduced. This reduction should manifest itself towards the limit of the distance at which the influence ceases. In the same way four tin sieves were taken and located at 40 mm. apart, and against each of them a small cylinder of gun-cotton was placed and the entire affair arranged on a board. 15 mm. in frout of the first sieve a similar cylinder was detonated. All of the cylinders detonated, but a progressive

diminution was observed in the excavations produced in the board below each cylinder. According to these facts the propagation by influence depends at the same time on the pressure acquired by the gas, and on the nature of the support. It is not even necessary that it should be rigid.

5. Finally, in operating under water at a depth of 1.30 meters, a charge of 5 kilograms of dynamite brought on an explosion of a charge of 4 kilograms, situated at a distance of 3 meters. The water then transmits the explosive shock, at least to a certain distance, as does a solid body. This transmission is so violent that the fish are killed in ponds without the sphere of a certain radius by the explosion of a dynamite cartridge, a process which is frequently employed to fish a body of water, but which is objectionable as depopulating the stream.

6. Similar experiments have been made by M. Abel, with compressed gun-cotton. According to his observations the explosion of the first block determines that of a series of similar blocks. The propagation under water has likewise been studied; the explosion of a torpedo charged with fulminating cotton caused the detonation of adjoining torpedoes placed within a certain radius of activity. The sudden pressures transmitted by the water when measured by means of the compression of lead at different distances, such as 2.50 m., 3.50 m., 4.50 m., 5.50 m; they go on decreasing, as would be expected. Besides, experiment has shown, that the relative position of the charge and of the "crusper" is of no consequence, which is unformidable to the principle of equal transmission in all directions of hydraulic pressures.

7. Explosions of fulminating substances belong to this same order of explosions by influence, which are rapidly propagated to a great number of caps. We have previously cited the explosion in the Rue Beranger. The experiments which M. Sarran made on that occasion showed that caps of the description which produced this catastrophe may be successively burned in a fire without giving rise to a general explosion; whereas the explosion of a few of these same caps, each containing 10 milligrammes of explosive material, if it is provoked by a rapid pressure, determines by influence the explosion of the adjoining packages, even when they are not contiguous and are situated at a distance of 15 centimeters apart. A general explosion may thus easily be produced by influence.

8. It follows then from these facts, and especially from the experiments made under water, that the explosions by influence are not due to a burning, properly so called, but to the transmission of a shock arising from the enormous and sudden pressures produced by the nitro-glycerine on gun-cotton.

Let us enlarge upon this explanation; it is the same fundamentally as that which we have already shown accounts for the influence of the shock which determines the direct detonation of explosive substances.

9. In an extremely rapid reaction, the pressures may approach to the limit which corresponds to the matter detonating in its own volume, and the commotion due to the sudden development of almost theoretical pressures can be propagated both through the ground and by supports as intermediary or through the air itself, projected *en masse*, as has been shown by the explosion of certain powder factories and of gun-cotton magazines, and even by some of the experiments with dynamite and compressed gun-cotton. The intensity of the shock propagated either by a column of air or by a liquid or solid mass, varies with the nature of the explosive body and its mode of inflammation; it is of greater violence according as the length of the chemical reaction is shorter and develops more gas, that is to say, a higher initial pressure, and more heat, that is to say, work, for the same weight of explosive material.

10. This transmission of a shock is conveyed better by solids than by liquids, better by liquids than by gases; with gases it is better, as they are more compressed. Through solids it is better propagated according to their degree of hardness, iron transmitting it better than earth, and hard ground better than ploughed soil.

All breaks of continuity in the transmitting material tend to weaken it, especially if a softer substance is inter-

posed. It is in this manner that the use of a tube as a receiver, made from a goose quill, stops the effect of mercury fulminate, while a tube or a capsule of copper transmits this effect in all its intensity.

The explosions by influence are the better propagated in a series of cartridges according as the envelope of the first detonating cartridge is the more resisting, which allows the gases to attain a greater pressure before the covering is destroyed.

The existence of an empty space, that is to say, filled with air, between the fulminate and the dynamite, on the other hand diminishes the violence of the shock transmitted, and in consequence that of the explosion; generally the effects of breaking powders are lessened when there is no contact.

11. To form a full conception of the transmission of sudden pressures which produce shock by the supporting medium, it is desirable to recall this general principle, in virtue of which in a homogeneous mass, pressures are transmitted equally in all directions and are the same on a small element of surface whatever its position. Detonations produced under water with gun-cotton show that this principle is equally applicable to the sudden pressures which produce the explosive phenomena. But it ceases to be true when one passes from one medium to another.

12. If the inert chemical matter which transmits the explosive movement is fixed in a given situation on the surface of the ground, or better, on the surface of the rail on which the first cartridge was placed, or better still, held by the pressure of a mass of deep water, in the midst of which the first detonation is produced, the propagation of the movement in this matter will hardly be able to take place except under the form of a wave of a purely physical order, and in consequence of a character essentially different from the first wave of a chemical and physical order, at the same time developed in the explosive body itself. This new wave propagates concussion away from the explosive center, all around it, and with an intensity which decreases inversely as the square of the distance. Even in the neighborhood of the center, the displacements of the molecules may break the cohesion of the mass and disperse it, or crush it by enlarging the chamber of explosion, if the operation is conducted in a cavity. But a very short distance, and of which the magnitude depends on the elasticity of the surrounding medium, these movements, confused at the beginning, regulate themselves in order to produce a wave, properly so called. Characterized by compressions and sudden deformations of the material, the amplitude of these oscillations depend upon the magnitude of the initial impulse. They move with a very great rapidity, and preserve their regularity up to the point where the medium is broken. Then these compressions and sudden deformations change their nature and are transformed into a movement of impulse, that is to say, that they reproduce the shock. If then they act on a new cartridge they may determine its explosion; the shock will be otherwise weakened by the distance, and in consequence the character of the explosion may be modified. The effects diminish in this manner up to a certain point, from which the explosion ceases to produce itself.

When this occurs on a second cartridge the same series of effects will be produced from the second to the third cartridge; but they depend on the character of the explosion of the second cartridge. And thus it goes on.

13. Such is the theory that appears to me to explain explosives by influence and the phenomena which accompany them. It depends, definitely, on the production of two orders of waves; one series represents the explosive waves, properly so called, developed in the midst of the matter which detonates, and consisting of a continual reproduction of the transformation of the chemical actions into caloric and mechanical action, which transmits the shock to the support and to the contiguous bodies; the other is a purely mechanical and physical series, and which also transmits the sudden pressures all round the center of the concussion to the adjoining bodies, and by a singular circumstance to a new mass of explosive material.

14. A theory differing from this was originally proposed by Abel; that is to say, the theory of *Synchronous vibrations*, to which we shall now direct our atten-

tion. According to this English savant, the originating cause of the detonation of an explosive lies in the synchronism, between the vibrations produced by the body which provokes the detonation, and those which the first body would produce in detonating, precisely as a violin string resounds at a distance in unison with another vibrating chord.

Prof. Abel has cited the following facts in support of his theory. To begin with, the detonators appear to differ with each variety of explosive. For instance, nitrogen iodide so susceptible to shock or friction, cannot cause the detonation of compressed gun-cotton. Nitrogen chloride, so easily exploded will not produce the same detonation, except when ten times the weight of the necessary fulminate is used. In the same way nitro-glycerine will not produce a detonation of gun-cotton in sheets on which is placed the envelope in which it is contained. In this way nitro-glycerine up to 23.3 grains can be detonated without success. On the other hand, the inverse influence is proved; 7.75 grains of compressed gun-cotton having caused the detonation at a distance of 25 mm. of nitro-glycerine wrapped up in an envelope of thin sheet iron. A cap filled with a mixture of potassium ferrocyanide and potassium chlorate, similarly, (according to Brown) will not detonate gun-cotton. Finally, a cap consisting of a mixture of mercury fulminate and potassium chlorate, should be used of much heavier weight than if it was filled with the pure fulminate, (according to Franzl). Nevertheless, the heat given off by the same weight is greater by one-fifth than that with the first mixture.

15. Messrs. Chapman and Pellet have brought to the support of this ingenious hypothesis the following experiments: they attached to the strings of a double bass particles of nitrogen iodide, a substance which detonates on the slightest friction. Then they made the strings of a similar instrument vibrate at a short distance off; a detonation was produced, but only for sounds higher than a certain note, which corresponds to 60 vibrations per second. They also took two conjugate parabolic mirrors placed 2.5 meters apart and they arranged along the line of foci at different points several drops of nitro-glycerine or of nitrogen-iodide, these they detonated at one of the foci a large drop of nitro-glycerine; they observed that the explosive substances placed in the conjugated foci detonated in unison to the exclusion of the same substances placed at other points. A layer of lamp-black placed on the surface of the mirrors was designed to prevent the reflection and the concentration of the calorific rays.

16. As yet none of the experiments appear to me to be conclusive, and several of them seem even to be formally opposed to the theory. We shall begin by observing that the speciality of a given musical note, capable of determining each variety of explosion, has never been established; it is only below a certain note that the effects cease to be produced while they take place by preference and whatever the explosive bodies may be by the action of the most acute notes. Besides, these effects cease to produce themselves at distances which are incomparably less than the resources of the chords in unison, which goes to prove that the detonations are functions of the intensity of the mechanical action, rather than of the character itself of the determining vibration. Similarly, the detonation ceases to be produced when the weight of the detonator is too slight, and in consequence when the living force of the shock is weakened. Nevertheless, the special vibratory note which determines the explosions should always remain the same. For instance, cartridges filled with 75 per cent. of dynamite cease to detonate when the capsule contains a weight of fulminate less than 0.2 grains; the detonation not being certain in all cases except by the regulation weight of one grain. This confirms the existence of a direct relation between the character of the detonation and the intensity of the shock produced by one and the same detonator.

If it is true that gun-cotton will cause the nitro-glycerine to detonate in consequence of the synchromism of the vibration communicated, then we do not understand why the reciprocal action does not take place, while the absence of reciprocity can be easily explained by the difference of the structure of the two substances which play so important a part in the transformation of the living force into work.

17. This same diversity of structure and the modifications which it introduces into the transmission of the phenomena of shock and with the transformation of mechanical energy into caloric energy, may be called upon to explain the facts observed by Abel.

The difference between the energy of pure fulminate and of the fulminate mixed with potassium chlorate is no less easily explained; the shock produced by the first body being sharper on account of the absence of all dissociation of the product, which is no other than carbon monoxide, this absence should be opposed to the dissociation of carbon dioxide formed in the second case. Perhaps, also, the formation of potassium chloride disseminated through the gas produced with the aid of potassium chlorate weakens the shock, just the same as silicon does in the case of dynamite.

18. All the effects observed with nitrogen-iodide may be explained by the vibration of the supports and by the effects of rubbing which results therefrom, this substance being particularly sensitive to friction.

19. The experiment with the conjugate mirrors may also be easily explained by the concentration in the foci of the movements of the air, and therefore of the mechanical effects which result.

20. Besides, M. Lambert has proved by experiments made for the commission of explosive substances, that the explosion of dynamite cartridges being produced in tubes of cast iron of large diameter, it does not appear to have, from the standpoint of detonations provoked by influence, any difference between the vibratory motions and nodes characteristic of the tube.

21. Desiring to clear up this entire question by removing it from the influence of the supports and of the diversity of cohesion and physical structure of solid explosive substances, I undertook a series of special experiments on the chemical stability of matter in a sonorous vibration and especially on that of gaseous bodies such as ozone or hydrogen, arsenide or liquids, such as hydrogen feroxide, or persulphuric acid, all of the bodies being selected from among those which decompose or change spontaneously, and that at ordinary temperatures with the disengagement of heat, precisely similar to explosive substances. The description of these experiments may be found in the *Comptes Rendus* or in the *Revue Scientifique*, May, 1880.

They lead to the conclusion, that substances which are transformable with the disengagement of heat, are stable under the influence of sound waves, while they are decomposed under the influence of ethereal vibrations. This diversity in the mode of action of the two classes of vibrations is not surprising when we consider that the sharpest sonorous vibrations are incomparably slower than the luminous or calorific vibrations.

22. Nevertheless, it appears certain that the propagation of explosions by influence is not made in virtue of an undulatory movement, a complex movement of a chemical and physical order in the midst of the explosive substance which is decomposed, whilst it is purely physical in the midst of intermediary substances which suffer no decomposition. But that which distinguishes this sort of movement of the vibrations, properly so called, is, first of all, its extreme intensity, that is to say, the magnitude of the living force which it transmits; it is also the unique character of the explosive wave which is propagated in opposition to the multiplicity of successive sonorous waves. Finally it is essential to observe that the explosive material does not detonate because it transmits the movement, but on the contrary because it arrests, and because it transforms on the spot the mechanical energy into calorific energy, capable of suddenly raising the temperature of the substance up to the degree which will produce its decomposition.

ANEMOMETRIC OBSERVATIONS.—M. Domojiroff continues to publish in the *Izvestia* of the Russian Geographical Society his anemometric observations on board the clipper "Djighit." In June, 1881, during the cruise from the Zond Strait to the Seychelles Islands, he met mostly with south-east winds, the velocity of which varied from 3 to 7.5 meters per second, with one exception, on June 9th, when it reached 15 meters. On the cruise from the Seychelles to Aden, from June 25th to 30th, the wind was mostly south-west, and varied from 5 to 12.7, reaching 14.3 meters per second on June 29th.

WHO DISCOVERED GUNPOWDER?

A HISTORICAL CHAT.

By KARL BRAUN, Wiesbaden, Leipzig.—Nord und Süd, Juni, 1883.

Translated by LIEUT. JOHN P. WISSER.

THE question, "Who discovered gunpowder?" is usually answered to-day *unisono:*

"Berthold Schwarz, the Freiburg monk."

So our youth has been taught for two generations, and this is quite enough to make any doubt of this assumed fact appear as idle folly.

Nevertheless doubt is justified. The contemporaneous writers, the authors of the middle and of the latter half of the fourteenth century, knew nothing of the discovery of the monk of Freiburg. The name of Berthold Schwarz is first mentioned long after "Büchsen" and "Katzen" (small cannon or mortars) were used in firing, and after a "Katzenstadl," *i. e.*, a gun-foundry, as well as an arsenal existed in Augsburg, for instance.

But even those who grant Schwarz the honor of being the first to make use of the preparation of gunpowder in Germany, and to spread the knowledge of its use, deny him part of the merit of the discovery. They assert that he too belongs to the great number of those "who did *not* discover gunpowder;" at all events he could not have taken out a patent on his "invention," for it had been in use for centuries. The Chinese had been long acquainted with it; traces of it are found among the Saracens and the Byzantines; it may be assumed, say they, that the discovery is derived from the Chinese, and has passed by various, no longer accurately determinable, steps, to the Byzantines, and through them has arrived in Germany; although the Byzantine or "Greek fire" is not identical with modern gunpowder, it is of earlier date, and the latter bears the same relation to the former that an amendment bears to the principal motion, or an additional or improvement pattern to the main patent.

Occupied with these doubts, I find in the "Chronicles of Augsburg," composed by the learned Clemens Jager, about the middle of the sixteenth century, the notice that *a Jew, named Typsiles, discovered gunpowder in the year 1353, in Augsburg, and from Augsburg the preparation of gunpowder, its application to military purposes, and the manufacture of fire-arms, spread throughout Germany and over the rest of Europe.*

True, the chronicler Clemens Jager, writes two hundred years after the discovery and the propagation of gunpowder manufacture in Europe, and cannot therefore speak from personal observation or the observation of his contemporaries. But the same is true of the warranters and witnesses of the patent of the monk of Freiburg. Clemens Jager is, however, to be regarded as an earnest and authentic writer, who has studied his sources carefully. We are compelled to believe that, to make such an assertion with such apodictic certainty, he must have had his good sources and grounds therefor, and that he could assume belief and agreement in his assertion from his fellow-citizens in Augsburg, who were acquainted with his sources, and instructed by the traditions of their forefathers on the subject. Indeed, his statement not only remained uncontradicted at the time, but was confirmed and repeated by other chroniclers and other authors of later date.

We may therefore assume as authentic that it was believed in Augsburg, in the sixteenth century, that the discovery or re-discovery of gunpowder by the said Typsiles took place within the walls of that good city.

I acknowledge that this view is founded on a legend as well as that which asserts the authorship of Berthold Schwarz. In this respect one has not much preference over the other. We also know little more of Schwarz than of Typsiles in both cases we must be content with the mere names.

But here there is nevertheless a slight difference. "Schwarz" belongs to the names which are so common that they hardly bear the stamp of individuality. Schwarz is a name like Brown or White, like Smith or Jones, like Miller or Baker.

Typsiles, on the contrary, has a meaning. The name is not of Jewish, but of Greek origin, when we consider Typto or Psilos, or regard it as a compound of the two, or of two similar words.

The name points to the Levant, to the Byzantine empire—to Constantinople, which at that time not yet conquered from the Turks, had still an active inter course with the West; we find, for instance, Byzantine comes everywhere, from Hungary and Roumania to Denmark and Sweden, and thence to Portugal and Spain. The old German shrines of relics are of Byzantine origin. So also the old imperial crowns. And the Hungarian king's crown, so celebrated for its age and adventures (it was several times sold, stolen, pawned, conquered, robbed, hidden, and yet always reproduced), and regarded by every good Hungarian as sacred, is of Byzantine origin.

It is a fact that the Byzantines possessed an explosive substance closely related to modern gunpowder, as it came into use in the middle of the fourteenth century in Germany, and middle and Western Europe.

These circumstances lead us to the conjecture that the said Typsiles, be he of Jewish, or Greek Catholic, or Roman Catholic confession—for faith has nothing to do with gunpowder—came from the Orient, and brought thence a knowledge of the preparation of Greek fire into the free imperial city of Augsburg. the metropolis then of the Alemannic countries in Germany, where, by modifications of the technical methods employed, he effected the preparation of our gunpowder.

I do not intend to write an account of the Greek fire, or the science of gunnery in Constantinople, which passed from the Byzantines to the Turks (as did, for instance, the dome of the churches, and much else), but only, *en passnnt*, to insert two interesting notices.

The "Greek fire" played its part on into the nineteenth century.

During the Greek war for independence in the twenties, the Greeks obtained only occasional successes by land, and these did not prove to be lasting. The separate bands of the armatoli, klephts and palikari, brave as they were, soon dispersed again. The truly decisive triumphs of more permanent effect were gained at sea, where a Miavlis and a Sachturis delivered murderous battle to the fleets of Chosren and Ibrahim; and here it was that the activity of the Greeks triumphed over the lethargy of the Turks, the small vessels of the Greeks, so capable of manœuvering, over the colossal, unwieldy and heavy vessels of the Turks; *and principally by means of fire-ships and the Greek fire.*

These small fire-ships, furnished with this combustible, each manned by nine, or at most twelve men, swarmed about the large Turkish ships, surrounded them on all sides and endeavored to deprive them of wind. The Greeks were familiar with the seas and coasts, those of the mainland as well as those of the innumerable islands, which latter had furnished the trained mariners, men of bravery and skill, inured to the perils of war and the sea, whose wants were so few that a handful of black olives sufficed for a day's subsistence. They were versed in the wind and weather of these seas, and could anticipate their character for several days, so as to prepare combined plans of operations in advance. The Turks, on the contrary, generally rode at anchor. "To anchor suits best the believers in Fatalism," (Mouiller convient aux adeptes du Fatalism) says the French Vice-Admiral Jurien de la Gravière, in his highly interesting contribution to the history of the Orient, from 1815 to 1830, which he has furnished in his book "La Station du Levant."

When the Turkish ships, finding themselves surrounded by the Greek fire ships, overcoming their fatalistic lethargy, finally put themselves in motion, it was generally too late to escape them by a precisely executed manœuvre. The fire-ship knew how to attach itself and its fire—its Greek fire—which burned on and exploded even under water, so skillfully that it could not be gotten rid of. The nine or twelve men in the fire-ship pulled rapidly out of danger in a light boat, w e the Greek fire blew the Turkish

vessel into the air, or at least tore open a breach of several meters in extent, and thereby usually succeeded in sinking it.

In our torpedoes and torpedo-boats we observe a new form and application of the Greek fire-ships and Greek fire, which has possibly entirely changed naval warfare. At all events the above-mentioned Jurien de la Gravière thought it possible to predict as much. Meanwhile the Germans have every reason to be grateful for the invention of torpedoes, for in 1870 they successfully protected and defended their coasts and seaports.

So much for the notice in regard to *the history of the Greek fire.*

The second notice relates to *the artillery of the Turks.* In the palmiest days of the Turkish Empire, in the fifteenth and sixteenth centuries, the Turkish army excelled in cavalry and artillery. As early as the fifteenth century a gun-foundry existed in Constantinople. In Turkish it is called *top-hané.* In the ear of the Turk the cannon shot does not sound as in our own, "bang," but "top." *Top* is the gun, and *hané* the house. Hence, "gun-house"; and this is precisely what "Katzenstadl" signifies in Augsburg. In the sixteenth century this gun-foundry, this *top-hané,* lying in the suburb Pera, enjoyed an extraordinary celebrity, and the writers of the day (the Genoese Giovanni Antonio Menavino, for instance) do not fail to add to their notice of this gun-foundry, that they were Greek Jews or Jewish Greeks who conducted the entire establishment, namely, the casting of cannon and the preparation of gunpowder, thus furnishing the elements of war and destruction to the hereditary and arch-fiend of Christendom, as the Turks were called, although then and thereafter in alliance, or at least in most cordial harmony, with the "most Christian King" of France. Here certain remarks are pertinent, which I hesitate to communicate. They may be oil on the flames of our anti-semitics of to-day. At all events these remarks cast a peculiar, even somewhat comic, retrospective light on the fact that it was also, as Clemens Jäger informs us, a Greek "Jew named Typsiles," who furnished the Christians of the West their elements of destruction and war to be used against the Mohammedans of the East.

The subject is therefore compensated.

Let us return, after the communication of these Greco-Turkish notices, from the East to the West, to Germany, to Augsburg. That this metropolis of the Alemanni, like Nürmberg, the metropolis of the Franks, stood then, in the fourteenth century, on the pinnacle of arts and manufactures in Germany, is an indisputable fact. Nürmberg was celebrated for the discovery of painting on glass, Augsburg for that of linen or rag paper (in contradistinction to the old parchment or the East Indian cotton paper). Even this claim is contested by other German cities—Ravensburg, for instance, which can produce a register of the year 1324 written on rag paper, and a linen paper mill of the year 1412. But the claims of Augsburg rest on older documents, namely—city accounts of the year 1320, undoubtedly genuine, and written on linen paper. It also possesses such a document of 1330, and many from 1360 on. In short, there is no doubt that Germany can produce the oldest documents on linen paper—older than those produced by Spain and Italy—and that, among the competing German cities it is Augsburg again which contains the oldest of these possessions. The importance of this discovery is apparent when it is remembered that the art of printing would not have spread so rapidly so soon after its discovery, had not linen paper, which surpasses parchment paper in cheapness and cotton paper in durability, already existed.

This same Augsburg, which rejoices in the oldest linen paper, rejoices also in the oldest cannon, *i. e.,* machines from which, *by means of gunpowder,* balls were fired at the enemy. These were then called "Katzen," or "Büchsen" (generally written "Puchsen," "Buchsen," or "Pugxen"). At first they were of wood with iron hoops, and threw stone balls. Augsburg made use of such machines as early as 1372, in the war against the Bavarian dukes. This is attested by the historian Adelzreiter, and confirmed by the city accounts, in which it may also be seen that the gunpowder manufactured by the city was made from saltpetre. In the city accounts of 1377 "gropze Büchsen," *large* guns, which the city ordered to be *cast,* are already mentioned, hence *metallic* cannon. There also existed a "Büchsen-meister," or a master-gunner, appointed by the city.

I select the following extracts from a valuable little paper on "Augsburg and its Former Industries," long ago out of print and forgotten, written and published, under commission from the city, by the industrious city recorder, Theodore Herberger, in 1852, on the occasion of the exposition of arts and manufactures of the Bavarian district of Suabia and Neuberg, which is based upon an accurate study of the archives there, namely, the city accounts.

In 1371 the city had expenditures "for saltpetre for the guns," for saltpetre for the manufacture of powder for the guns. In the account 20 guns are mentioned as being used in firing; moreover, "Trinkgelder" (pour boire) for the vassals who served these guns. The expenditures for the wooden frames on which the guns were supported are reckoned in the account. A year later, 1372, 400 shot were cast for the guns; lead "for casting" occurs in the account, saltpetre and "wilder schwefel" (wild sulphur) for gunpowder. One year later, 1373, the expenditure for copper, lead, "and other material" is reckoned in the account "for 4 guns. Another year later there occurs in the accounts an item "for a mortar, in which powder for cannon is pulverized." Many such and similar items may be cited to show how far advanced Augsburg was when the art of firing with heavy guns began. Master Walther, the master-gunner, was not only paid, in 1373, the uncommonly large sum of 160fl., but also received a special present in cloth for constructing the guns ordered, and inspecting the preparation of gunpowder in the court of a "canon of St. Moriz." An unusual number of large cannon was manufactured, according to the accounts, in the years 1410 and 1414, and in the year 1416 the master-gunner. Ott, who was also employed to cast bells in foreign cities, cast several large pieces. All this proves the early date of an immense trade in this department. An especially remarkable man appears in Augsburg in the year 1436. Master Heinrich Roggenburger, the master-gunner. His office is more particularly "the casting of guns, large and small," and the firing of them "as dexterously as has ever been seen;" he can also prepare the powder therefor. Besides, he is a man remarkably well versed in the technicalities of his art in other respects also, and in his letter of admission he is recommended for the following qualities: He can "make cast and projectile apparatus, large and small, the like of which was never seen in German lands, for this apparatus stands still after the throw, without moving or altering its position, and not requiring to be bound or held;" these machines throw masses of five or six hundred-weight; besides, he makes lifting-machines, by means of which a hundred hundred-weight may be lifted from or upon a wagon; also shields for guns and war chariots, and bridges which may be carried over land and laid over ditches or running water. Moreover, he understands the building of houses and towers, water-mills, windmills and horse-mills, and can make cast, earthen and wooden water conduits to supply the water of wells to hill and valley. Roggenburger received a yearly salary of 110 fl. In the year 1502 the town had a foundry of its own built, which was called Katzenstadl. Here, according to the account of his contemporary, Clemens Sender, Niklas Oberacker cast one hundred metallic pieces and a mortar; among the larger pieces were several forty feet in length. The most noted of all the gun-founders of Augsburg was Gregor Löffler. He was much occupied, not only in Augsburg, but also in foreign countries. In the year 1529 the Government called him to Innsbruck. In this year and in 1537 he had orders to recast all the old pieces which the Emperor and King Ferdinand had in the Tyrol. Among the newly cast cannon were "Karthaunen," capable of firing a shot of an hundred-weight. This work gained for him such approbation that he was entrusted with casting the statues designed to decorate the tomb of the Emperor Maximilian.

Thus far the recorder Herberger. The statues in Augsburg cast by Gregor Löffel are, nevertheless, not identical with those colossal life-statues which now surround the tomb of the "last of the knights" in the Franciscan church at Innsbruck, the authors of which were the brothers Stephen and Melchior Godl. The statues of Löffler, representing various saints, twenty three in number, are found in the same church, in the so-called "Silver Chapel," on the south wall.

I will not further expand this chapter

on Augsburg gunpowder and Augsburg ordnance; whoever desires to pursue the subject further, him I refer to the Augsburg chronicles of the fourteenth and fifteenth centuries, published by Professor Karl Mayer.

I hasten to conclude. I am aware that this unassuming chat does not solve the problem, but only brings us a trifle nearer the solution. I only desired to instigate doubt and investigation.

THE RATIONAL FORMULA FOR PILLARS APPLIED TO TESTS MADE WITH THE UNITED STATES GOVERNMENT MACHINE AT WATERTOWN, MASS.

By JOHN D. CREHORE.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

In an article entitled "*A new rational formula for pillars*," which appeared in this Magazine for December, 1879, I gave this formula:

$$Q=\frac{P}{S}=\frac{C}{1+\frac{Cl^2}{m\pi^2 Er^2}} \quad . \quad . \quad . \quad (1)$$

where $m=1$, 4, or 2.28 according as we consider neither, both, or one only of the pillar's ends fixed.

l=length of pillar.

r=least radius of gyration of cross-section, both in inches.

S=area of cross-section in square inches.

E=modulus of elasticity in pounds per square inch.

C=crushing strength of standard specimen in pounds per square inch.

P=breaking weight applied at the end of the pillar and in the line of its axis before deflection, in pounds.

Q=breaking weight in pounds per square inch of cross-section.

Certain conditions upon which the formula depends were pointed out and insisted upon for its proper application to finished pillars. These conditions are:

1st. The pillar must deflect or bend before it breaks.

2d. When $m=1$, the points of bearing at both ends of the pillar must be in the axis, and the curvature must be single.

3d. When $m=4$, the line of the axis of the straight, undeflected pillar must be tangent to the curved axis under load, at both ends, and there must be two points of contrary flexure.

4th. For values of m lying between 1 and 4, there will be a blending of parts of the 2d and 3d conditions.

5th. When $m<1$, it is inferred that the line of pressure is not coincident with the axis; and,

6th. When $m>4$, it is to be presumed that the deflection is, for some reason, less than the normal deflection.

It is not the object of the present paper to advocate the adoption of this formula; but, rather, to determine, as far as may be done from the experiments cited, what values are to be assigned to m in a formula based on theoretical considerations, in order that it may apply with reasonable approximation to the compressed members of actual structures, seldom, if ever, fulfilling the conditions imposed upon the formula. Of course the formula, called rational, will become empirical by such determination of the value of m; but this is of no importance

if thereby it shall become also more trustworthy and useful.

Anyone who has ever undertaken the task of adapting a formula to the results of a large number of tests, knows very well how much it is like finding the "laws of disorder." Illustrating this point, Mr. Benjamin Baker has recently said, "taking the mean results of a large number of experiments, the influence of length between the practical limits of 15 to 20 diameters would just be appreciable; but if only a few experiments were compared, the deduction might be drawn that lengthening a column or rounding its ends, increased rather than reduced its strength. Thus, one steel tube 20 diameters in length, tested by the author, bore 22 tons per square inch, whilst a similar tube of half the length bore only 19.2 tons. Again, a round-ended column, 20 diameters in length, bore 19.3 tons per square inch, whilst a flat ended one failed with 18 tons."

Similar anomalies will be found in the tables given below; and no definite and certain explanation of them will be attempted here, for the reason that there is not sufficient knowledge of the material, and the manufacture, and the absolute conditions of the columns, to warrant such an attempt; although, in a few cases, the data at hand seem to throw light on the eccentric behavior reported.

The complete and final determination of the value of m, cannot be made from the few experiments at hand, and with calculated values of C; but, since many such columns are now in existing bridges, and probably many more like them will be placed in future bridges, with no better determination of C, a value for m, though it be only provisional, seems desirable.

The latticed columns of the Detroit Bridge and Iron Company were built of two wrought-iron rolled channels with their flanges outward, and lattice bars riveted to each flange at intervals of 18 inches for the six-inch and eight-inch channels, and at intervals of 22 inches for the ten-inch and twelve-inch channels. These lattice bars had a cross-section ranging from 2 by $\frac{1}{4}$ to 2 by $\frac{3}{8}$ inches, and were placed so as to "break joints" on the opposite sides of the column; thus leaving only 9 or 11 inches of wholly unsupported channel length.

The pin holes were $3\frac{1}{2}$ inches in diameter when the channels were spaced 8 inches apart, and were 3 inches in diameter for channels spaced 6 inches. From Colonel Laidley's Report for 1881, I quote:

"All columns with pin bearings tested with pins in a vertical position.

"Columns over 12 feet in length, supported at the middle by a counter-weight of half the weight of the column.

"Strains gradually applied; the time of the test occupying $1\frac{1}{2}$ hours, unless otherwise stated in the detailed notes of the test.

"Sectional areas of channel bars, computed from the weights of the bars before the rivet holes and pin holes were made; calling the sectional area in square inches one-tenth the weight of the bar in pounds per yard.

"Horizontal and vertical deflections measured at the middle of the columns.

"The compressions measured within the gauged length, a distance laid off along the middle of the upper channel bar, always taking a gauged length less than the distance between the eye plates, to avoid any disturbance of the channel-bar webs the eye plates might occasion.

"Many columns are found to have the pin holes bored out of parallel and not at right angles to the axis of the column. In such cases thin brass packing was placed between the bolsters carrying the pins of the columns and the faces of the compression platforms of the testing machine.

"When such packing was used, the amount in thickness is recorded, showing what was necessary to secure a good bearing for the ends.

"The eye plates, riveted to the webs of the channel bars, in some cases slipped, allowing the pin hole in the web to elongate without disturbing the holes in the riveted plate. The slipping, probably, took place when the friction of the rivet heads was overcome."

From equation (1) we derive

$$C = \frac{Q}{1 - \frac{Ql^2}{m\pi^2 E r^2}}, \quad . \quad . \quad . \quad (2).$$

and $$m=\frac{CQl^2}{(C-Q)\pi^2Er^2} \quad . \quad . \quad . \quad (3).$$

Take $$m_0=\frac{300\,m}{l}=\frac{300\,CQl}{(C-Q)\pi^2Er^2} \quad . \quad . \quad (4).$$

Modulus of elasticity,

$$E=\frac{P_1l_1}{S\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

where P_1=the pressure which shortens the pillar by the length, λ in pounds.

l_1=gauged length in inches.

In the accompanying tables all values of l, l_1, P, P_1, Q, λ and S, are taken from the published reports. Q, in Table I, is a mean of two of its values in Table II, for the given length of column.

Table I is given to show the probable effect of thickness of flange where it meets the web, on the strength of the channel used as a strut. It there appears, from the few examples cited, that the strength of a channel strut is roughly proportional to said thickness, allowance being made for length and radius of gyration. Should this law hold generally, it is worthy the attention of manufacturers.

In computing the radius of gyration, r, for columns formed of two latticed channels, the section of each flange has been treated as a trapezoid instead of a rectangle; thus making r, in Table III, a little smaller and nearer correct than by the ordinary method.

In Table II, E is derived from the compression of columns having two channels of the given section, in Table III, and computed by equation (5).

C is obtained from equation (2), assuming $m=4$.

The greatest value of C, in Table II, found for channel bars of each given size, is used in computing m, by formula (3), for Table III.

The deflections in Table III, are those due to the load next before the ultimate.

In Table IV, C=the mean of the two greatest values of Q, for each year 1879 and 1881.

From equations (3) and (4) are found the values of m and m_0.

The factor $\frac{300}{l}$, (l being in inches) is introduced into m, to show that the resulting m_0 is more uniform than m, and is more nearly within the normal range of m from 1 to 4, in case of these examples of Phoenix and Kellogg columns.

By comparing the cases giving the anomalous values of m, that is, values greater than 4 and less than 1, with normal cases of the same ratio ($l \div r$), we see that the values of Q do not keep pace with those of $l \div r$. For instance, in Table III, observe Nos. 1229 and 1230; Nos. 117, 118, 119, 120; Nos. 24 and 25; Nos. 4, 5, 6, 26; Nos. 1109, 17, 15, 16; and Nos. 468, 469, 470. In Table IV, Nos. 12 to 18; and in Table V, Nos. 3 to 7.

Now, since the ends of a pillar cannot be more than fully "fixed," and fixed ends require $m=4$, it seems to follow that the greater values of m, belong to the cases where the pillar is so well conditioned that its deflection is small up to the breaking point. And, where $m<1$, there is manifestly some irregularity in the construction of the pillar; as, for instance, No. 469 of Table III, of which test the record says: "About .035 inch packing at top of south bolster required to make good bearing. Column not straight; lower channel concave .26 inch; upper channel convex .22 inch; measuring at middle and from outside; straight in other direction." And No. 326 of Table IV, where m is small for a flat-end column, has this record: "There was a space of .22 inch between the upper side of one end and the compression platform, and the same space at the under side of the other end, while the column was under a strain of 10,000 pounds. It required about 80 per cent. of the ultimate crippling load to bring the ends to bearing all around the web and flanges."

Another source of irregularity of m for pin-end columns, is the fact that the deflection is seldom wholly normal to the plane of the pins; and hence the least radius of gyration is not the one to be used in these cases which have an inherent bias to deflect out of the plane of the least circle of gyration. This bias may be partly due to a difference in the size of the two channels composing the column. To illustrate this, we observe that of the 45 cases in Table III, where the two channels were of unequal sections, 27 deflected in the direction of the larger channel, and 18 in that of the smaller; showing that in $\frac{3}{5}$ of the cases the smaller channel yielded first to the pressure.

It might appear that no part of a channel *in a column*, sustains so great a load as the same unsupported length sustains when not built in. This is merely apparent; for, when in the column, the mean strength of the two channels is observed, while in fact one of them is generally strained far beyond this mean.

Evidently more tests are needed, using many columns apparently identical, in order that the most probable values of the constants may be found.

TABLE I.

CHANNEL BARS.

Nominal size, depth, and length. Inches.	Actual depth. Inches.	Width of flange. Inches.	Thickness of flange.		Nominal thickness of web. Inches.	Nominal area. Square inches.	Ratio of length to radius of gyration. $l \div r$.	$\frac{tr}{\cdot}$	Mean value of Q, lbs.
			At web. Inches. t.	At edge. Inches.					
6	6.07	1.71	.43	.24	.22	2.33	12.504	.0358	42,493
8	8.08	2.02	.62	.25	.29	3.85	15.028	.0413	43,295
10	10.00	2.48	.39	.30	.33	4.78	15.251	.0256	35,080
12	12.06	3.03	.40	.38	.32	5.97	13.944	0287	37,240

TABLE II.

COMPRESSION TESTS OF CHANNEL BARS. FLAT ENDS.

No. of test.	Nominal size of bar. Inches.	Length l. Inches.	Sectional area S. Square inches.	Ultimate strength, Q lbs. per sq. inch.	Manner of failure.		$l \div r$.	$\frac{E}{1000}$	$m = 4$. C. lbs. per sq. inch.
					Buckled.	Deflected.			
1049	6	6	2.33	42,290	1		12.504	28,053	42,548
1050	"	6	2.33	42,695	1		12.504	28,053	42,958
1071	"	17.58	2.37	36,670		1	36.872	28,288	38,383
1072	"	17.70	2.23	37,000		1	36.298	31,762	38,763
1069	"	23.83	2.23	35,160		1	48.869	31,762	38,148
1070	"	23.90	2.37	32,660		1	50.128	26,144	35,481
1064	"	48	2.38	28,140		1	100.835	28,288	37,574
1051	8	8	3.85	42,780	1		15.028	26,801	43,146
1052	"	8	3.85	43,810	1		15.028	26,801	44,194
1068	"	17.90	3.73	35,280	1		33.199	25,501	36,821
1065	"	23.85	3.73	36,540		1	44.234	25,501	39,201
1066	"	23.85	3.73	35,410		1	44.234	26,680	37,903
1067	"	29.90	3.73	33,400		1	55.455	26,680	37,000
1063	"	48	3.73	30,620		1	89.025	26,680	39,787
1053	10	10	4.78	34,810	1		15.251	28,126	35,066
1054	"	10	4.78	35,350	1		15 251	28,126	35,614
1074	"	17.85	4.76	33,820	1		27.173	28,126	34,598
1075	"	23.90	5.04	35,080	1		37.326	25,286	36,888
1076	"	23.87	5.04	33,630	1		37.279	25,286	35,282
1073	"	29.90	4.76	34,050		1	45.517	28,126	36,360
1062	"	48	4.76	34,080		1	73.070	28,126	40,759
1055	12	12	5.97	37,240	1		13.944	27,872	37,487
1056	"	12	5.97	37,240	1		13.944	27 872	37,487
1079	"	17.84	5.95	36,590	1		20.691	27,872	37,119
1077	"	23.92	6.02	36 350	1		27 891	29,630	37,250
1078	"	23.87	6.02	37,040	1		27.897	29,630	37,971
1080	"	29.90	5.96	35,150	1		34.711	27,872	36,557
1061	"	48	6.19	36,040	1		56.630	27,176	40,219

TABLE III.

LATTICED COLUMNS BUILT BY THE DETROIT BRIDGE AND IRON COMPANY. NOS. 1059, 1060 HAVE FLAT ENDS; NOS. 1095, 1096 HAVE EACH ONE PIN END; ALL OTHERS HAVE TWO PIN ENDS, TESTED WITH PINS VERTICAL.

No. of test.	Size of Channels. Inches.	Length of Post. l inches.	Cross section. S sq. Inches.	Gauged length. l_1 inches.	P_1 = load giving $\frac{\lambda}{1000}$ lbs.	Compression due P_1. λ inches.	Penultimate deflection. Hor. D ins.	Penultimate deflection. Ver. D ins.	$\frac{E}{1000}$ lbs.	$l \div r$.	Q lbs.	C lbs.	m.
					CHANNELS 8 INCHES APART.								
1059	6	120	4.76	80	135	.0786	.02	.10	28,867	52.623	36,720	42,958	2.457
1060	"	"	4.67	"	120	.0745	.07	.05	27,593	52.292	35,330	"	1.998
1095	"	"	4.75	"	130	.0774	0.	.02	28,288	52.555	33,680	"	1.543
1096	"	"	4.58	"	"	.0890	−.03	−.04	25,514	51.903	33.800	"	1.696
1107	"	144	4.60	100	"	.1100	−.01	0.	25,692	62.478	34,740	"	2.796
1108	"	"	4.57	"	"	.1123	.10	.06	25,331	62.386	34.160	"	2.597
1	"	150	4.56	120	120	.1163	−.07	.08	27,153	64.938	35.880	"	3.427
2	"	"	4.74	"	"	.1162	.20	0.	26,144	65.611	32,380	"	2.194
1231	"	180	4 48	150	"	.1265	−.25	.10	31,762	77.553	33,820	"	3.122
1232	"	"	4.56	"	"	.1396	−.08	−.02	28,276	77.925	34,540	"	3.835
1229	"	210	4.66	180	125	.1712	.05	.06	28,203	91.455	32 750	"	4.141
1230	"	"	4.74	"	"	.1637	−.21	.02	28,997	91.855	31,120	"	3.337
1117	"	240	4.66	200	120	.1955	−.41	−.02	26,344	104.520	29.180	"	3.822
1118	"	"	4.63	"	"	.1850	.18	−.12	28,019	104.340	30,990	"	4.379
1119	"	270	4.57	230	"	.2155	.24	.10	28,025	116.975	30,590	"	5.256
1120	"	"	4.66	"	"	.2206	−.06	−.01	26,848	117.585	31,050	"	5.845
1121	"	300	4.71	260	110	.2090	−.37	0.	29,054	131 022	23 350	"	3.063
1122	"	"	4.63	"	"	.2250	.32	−.09	27 454	130.420	25,360	"	3 886
20	"	330	4.69	280	80	.1571	.28	.04	30,401	143 970	21.850	"	3.070
21	"	"	4.67	"	"	.1616	−.40	0.	29,681	143.700	20,810	"	2.849
18	"	360	4.70	320	65	.1386	.50	.06	31.930	157.510	14,740	"	1.758
19	"	"	4.73	"	"	.1370	−.57	.10	32,098	157.090	15,900	"	1.966
1111	8	160	7.52	120	220	.1320	−.02	.04	26,598	53.229	34,810	44,194	1.769
1112	"	"	7.50	"	"	.1265	0.	.06	27 826	53 192	35,240	"	1.792
1113	"	200	7.48	150	"	.1730	.10	.04	25,502	66.443	33,970	"	2.724
1114	"	"	7.48	"	"	.1830	−.18	−.05	24,108	66.443	33,610	"	2.461
1115	"	240	7.55	200	"	.2461	−.14	−.23	23,681	79.995	32,610	"	3 406
1116	"	"	7.51	"	"	.2196	.25	.33	26.680	79.816	32,140	"	2.851
1123	"	280	7.99	240	"	.2320	.05	.10	28,484	94.604	32,230	"	3 790
1124	"	"	7.67	"	200	.2335	.10	.07	26,801	93.623	31,370	"	3.582
24	"	320	7.78	280	150	.1800	.10	.22	29.991	107.280	31,350	"	4.202
25	"	"	7.75	"	"	.1978	.04	−.07	28,857	107 290	27,850	"	3.043
22	"	360	7.81	320	100	.1376	.60	−.03	29,777	120.930	24,850	"	2.825
23	"	"	7.80	"	"	.1415	.03	−.02	28,993	120.900	26,920	"	3.518
13	10	150	9 68	100	250	.0971	0.	−.05	26.598	41.748	35 550	40,759	1.847
14	"	"	9.59	"	"	.0984	.03	.05	26 493	41.758	35,350	"	1.768
11	"	200	9 55	150	"	.1340	.20	.12	28.126	55.009	33,840	"	2.212
12	"	"	9.61	"	"	.1419	−.10	−.10	27.499	55.012	34,000	"	2.432
3	"	250	9.74	200	"	.1879	.02	−.20	27.320	69.675	33.880	"	3.614
4	"	"	9.81	"	"	.1752	.10	.10	29,036	69.778	33,660	"	3.283
5	"	300	10.04	250	"	.2175	−.17	−.30	28 621	84.163	34,130	"	5.262
6	"	"	10.00	"	"	.2466	.16	−.35	25,286	84 109	31,930	"	4.177
26	"	350	9.30	300	150	.1615	.22	0.	30,659	96.748	32,180	"	4.708
27	"	"	9.57	"	"	.1574	.57	−.19	29 875	97.166	29,380	"	3.370
1109	12	120	12.15	70	370	.0833	0.	.06	25,590	27.024	33,420	40,219	0.572
17	"	"	12.06	"	300	.0605	0.	0.	28,126	26.978	35,070	"	0.718
15	"	180	12.12	130	"	.1086	0.	.03	29,630	40.513	33,830	"	1.195
16	"	"	12.47	"	"	.1110	0.	.05	28,176	40.780	35,490	"	1.805
9	"	240	12.98	200	290	.1566	−.01	.04	30,916	53.872	34,360	"	2.272
10	"	"	12.34	"	320	.1951	0.	−.20	25,583	54.242	33,610	"	2.294
7	"	300	12.14	250	290	.2168	.19	−.05	27,537	67 343	32,940	"	3.037
8	"	"	11.91	"	"	.2184	.05	−.05	27,872	67.250	34,240	"	3.786
28	"	360	12.18	300	300	.2220	−.10	−.45	28,846	81.120	31,610	"	3.413
29	"	"	12.54	"	"	.2060	.30	.10	30,195	81.662	31,340	"	3.177

TABLE III.—(*Continued.*)

No. of test.	Size of Channels. Inches.	Length of Post. l inches.	Cross section. S sq. inches.	Gauged length. l_1 inches.	P_1 = load giving λ. $\frac{P_1}{1000}$ lbs.	Compression due P_1. λ inches.	Penultimate deflection. Hor. D ins.	Penultimate deflection. Ver. D ins.	$\frac{E}{1000}$ lbs.	$l \div r$.	Q lbs.	C lbs.	m.
					CHANNELS 6 INCHES APART.								
463	6	240	4.68	190.50	22	.149	.07	0.	28,128	102.240	25,000	42,958	2.252
464	"	300	4.68	239.50	12	.098	.03	.03	30,568	127.800	15,260	"	1.282
465	8	240	7.75	190.50	24	.173	.09	−.06	27,673	80.000	27,750	44,194	1.748
466	"	300	7.75	242.00	24	.189	.07	.12	30,732	100.000	26,000	"	2 082
467	10	240	9.19	182.00	26	.150	−.03	.22	31,547	65.785	29,980	40,759	1.576
468	"	300	9.19	235.00	26	.200	.12	.02	30,550	81.570	32,000	"	3.271
469	12	240	12 95	181.50	28	.162	0.	.46	31,370	54.114	28,970	40,219	.980
470	"	300	12.95	234.75	24	.203	−.23	−.05	27,754	66.944	30,000	"	1.901

Mean value of m for flat ends, Table III 2.227
" " " one pin end, Table III..... 1.620
" " " two pin ends, " 2.885

TABLE IV.

PHOENIX COLUMNS. FOUR SEGMENTS CIRCULAR. FLAT ENDS.

No. of test.	Length of column l. inches.	Cross section S. sq. ins.	Gauged length l_1 inches.	P_1 = the load giving λ. $\frac{P_1}{1000}$ lbs.	Compression due P_1 λ. inches.	$\frac{E}{1000}$ lbs.	$l \div r$.	Q. lbs.	C. lbs.	m.	$m_0 = \frac{300m}{l}$
1879.			$l=l_1$								
1	336	12.062		360	.368	27,250	115.860	35,150	57,215	4.57	4.08
2	"	12.180		402	.424	26,155	"	34,150	"	4.43	3.95
3	300	12.233		396	.356	27,280	103.690	35,270	"	3.67	3.67
4	"	12.100		396	.377	26,043	"	35,040	"	3.78	3.78
5	264	12.371		300	.240	26,455	91.243	35,570	"	3 00	3.41
6	"	12.511		300	.236	27,260	"	34,360	"	2.66	3.02
7	228	12.023		300	.198	28,732	78.801	35,365	"	2.03	2.67
8	"	12.087		300	.213	26,568	"	36,900	"	2.46	3.24
9	192	12.000		200	.120	27,842	66.317	36,580	"	1.62	2.54
10	"	12.000		200	.116	27,569	"	36,580	"	1 64	2 56
11	156	12.185		300	.142	27,048	53.916	36,857	"	1.13	2.17
12	"	12.069		200	.091	28,474	"	37,200	"	1.13	2.16
13	120	12.248		327	.120	26,660	41.474	36.480	"	.66	1.65
14	"	12.339		340	.126	26,303	"	36,396	"	.66	1.66
15	84	12.265		200	.054	25,365	29.032	38,157	"	.39	1.38
16	"	11 962		240	.067	25,155	"	43,300	"	.60	2 16
17	48	12.081		320	.051	24,930	16,590	49,500	"	.41	2.57
18	"	12.119		320	.046	27,553	"	51,240	"	.50	3.10
19	8	11.903		400	.020		2.765	57,130	"		
20	"	11.903		400	.016		"	57,300	"		
Mean	.	..	..	..	..	..	..	..	..	1.96	2.82
1881.											
325	30.000	11.610					10.368	56,070	54,435		
326	142.625	12.181					49 294	38,256	"	1.03	2.17
327	29.940	11.902	24.016	350	.0230	30,706	10.348	52,800	"		
31	372.000	11.430	300	250	.2146	30,576	128.570	31,150	"	4.00	3.21
32	372.000	11.310	300	250	.2196	30,197	128.570	32,700	"	4.54	3.66
33	378.000	11.660	300	250	.2175	29,573	130.643	31,180	"	4.26	3 38
34	378.000	11.580	300	250	.2170	29,846	130.643	32,220	"	4.57	3.63
Mean	..	..	..	..	..	..	..	..	..	3.68	3.21

Mean of all, in Table IV..... 2.33 2.90

TABLE V.

KELLOGG COLUMNS.—Two 10-inch channels, latticed with flanges turned inward. Webs about 11 inches apart, except No. 8, which has flanges turned outward, and webs 4.5 inches apart. Pin hole of No. 9 was 3.915 inches in diameter; other pin holes 3 inches in diameter. No. 8 reached the limit of the machine, viz., 800,000 lbs., or about 40,000 lbs. per square inch of section. C is here put equal to 40,000.

No. of test.	Condition of ends.	Length of column l. inches.	Cross-section S. sq. ins.	P_1 = the load giving λ. $\frac{P_1}{1000}$ lbs.	Compression due P_1 λ. inches.	$\frac{E}{1000}$	$l \div r$.	Q.	m.	$\frac{m_0}{300\,m} = \frac{}{l}$
1879.										
1	1 Pin....	285.3	19.244	450	.218	30,603	78.916	25,980	1.53	1.61
2	Flat.	333	19.447	400	.233	29,396	92.110	28,280	2.82	2.54
3	"	237	19.486	450	.165	33,171	65.554	30,740	1.74	2.21
4	1 Pin....	189	19.301	200	.057	34,359	52.267	25,130	.55	.86
5	"	141	19.561	200	.041	35,161	39.002	25,460	.31	.65
6	"	96	19.103	200	.043	23,375	26.554	30,080	.37	1.16
7	Flat....	48	19.362	200	.020	24,791	13.277	37,180	.38	2.37
8	"	10	20.322				2.766			
9	1 Pin....	264.5	19.244	200	.078	35,342	73.162	27,900	1.42	1.61
Mean.	..	..	..	..			..	..	1.14	1.63
1881.										
492	1 Pin....	260	17.650	459	.195	29,866	71.918	33,920	3.26	3.77

Mean of all, in Table V	1.37	1.86
" for flat ends, "	1.65	2.37
" " 1 pin end, "	1.24	1.61

RESISTANCE OF RAILWAY CURVES.

From "The Engineer."

IN following exactly the dictates of experience one treads a tolerably safe path; but it is hardly from engineers that we should expect so servile a course, though the expression "experience is a better teacher than theory" finds much favor with some engineers who ought to be able to lay claim to a more independent and progressive policy. It was used on Tuesday evening at the Institution of Civil Engineers as a sort of Promethean warning that one inclined to make a theoretical investigation should beware of any conclusions which would not be in perfect harmony with the received views of the practical man. Engineers as much as those of any profession, have had to acknowledge their ignorance on subjects which their practical experience had led them to prophecy upon against the projects of some little-known man of advanced ideas or theoretical knowledge. They should learn now that even many years of experience will not always guide them in pronouncing a hasty opinion on carefully thought out investigations by younger men of more advanced times. They should remember that it is not those who have steadily observed the beaten track that have commanded things to progess, though they have themselves been on the whole more substantially successful than the innovator. Even railway men of long experience cannot always safely predict that departure from the dictates of that experience must be disadvantageous.

A paper was read last week before the Institution on "Resistance on Railway Curves as an Element of Danger," and was discussed last Tuesday evening. The subject is an old one, but the author had

something new to say upon it; he approached it from a new point of view and in a most temperate manner. Usually it has been considered that the intensity of the pressure brought to bear against the flanges of railway wheels, and especially of leading wheels, in passing round curves, has depended, as one chief element, on the speed at which the curve is traversed; and a maximum velocity has been assumed in calculating the greatest derailing force to be counteracted by flanges. In considering the best form of flanges the same idea has been followed as in the article by Herr Wöhler, referred to in our impression for the 2d March, 1883; but the author of the paper above referred to, Mr. J. Mackenzie, showed that derailment is as likely at slow speeds as at higher velocities, and that the danger of derailment does not necessarily bear any relation to the curve radius. He has thus to appeal to some other cause than centrifugal tendency to account for the pressure against the flanges of the wheels on the outer part of the curve, and for the reason of the subsequent derailment. The active cause he finds in the adhesion of the wheels on the rails, which has to be overcome by the flanges, in correcting the tendency of pairs of wheels of the same diameter rigidly fixed to the same axle, to pursue a path tangential to the curve instead of following it. He says, "In the case of the wheel most likely to mount the rail, namely the outer leading wheel, this side pressure, at slow speeds, is principally caused by the resistance which the treads of the wheels oppose to the sliding motion which takes place in running round a curve." Neglecting the effect of the conical form given to the treads of wheels, it will be seen, as is well known, that of a pair of wheels running round a curve, either the inner one must slip backwards or the outer one forwards, and this slipping is done against the adhesive resistance of the wheels on the rails. Mr. Mackenzie puts it thus: "In order to cause these sliding motions, the outer leading wheel flange exerts against the rail a pressure sufficient to overcome the adhesion or friction of the treads of the wheels; this pressure being exerted directly on the leading wheels, and transmitted to the other wheels through the medium of the engine framing acting as a lever." This, it will be seen, is independent of the speed at which the engine is traversing the curve, except that the friction may be less at high speeds; and from it it follows that when the adhesion between the flange and the rail is greater than the weight upon the wheel the flange will rise and mount the rail. This would take place if the surfaces of contact were vertical, and probably takes place much more easily with the inclined flanges which are always used. Generally, it may be said that a wheel, with flanges approximately vertical, would be caused to mount the rail by a side pressure bearing the same proportion to the load on the wheel which the load bears to the adhesion, while the pressure required diminishes rapidly with the increase of the angle of the flange inclination. Any one who has traveled on a footplate, or watched an engine coming round a curve, will have observed the effect of this side pressure on the outer leading wheel flange, and the periodic forced slipping of the wheels resulting in the engine passing round the curve by a series of jerks transverse to the rail, indicating that the pressure against the flange of the outer leading wheel periodically increases until it is enough to cause that wheel to push the whole engine transversely. As pointed out by the author, "the point of contact between the flange and the rail being in advance of the center of the axle, the motion of the flange at that point is downwards, imparting a downward pressure to the rail, and an upward pressure to the wheel, so that when the flange adheres to the rail the wheel rises. Thus the pressure which would cause the flange to mount the rail is not that which, with the wheel at rest, would force it over the rail in opposition to friction as well as to gravitation, but the very much smaller pressure which, when the wheel is at rest and the tread raised slightly above the rail, would cause friction sufficient to prevent its falling into its place again." From what has been said it will be seen that the action described takes place on a curve of large radius just as it does on a small curve; but with equal slackness of gauge, a greater distance will be traversed in the large curve before an equal pressure is brought to bear on the outer leading wheel flange.

On the whole, it seems difficult to up-

set the author's argument, though in applying his reasoning in a calculation relating to any given six-wheeled engine on a curve of given radius, and taking a normal coefficient of adhesion, it would be found that engines ought more frequently to leave the rails; but in applying the theory the whole of the modifying conditions must be taken into account, including the tractive force exerted, superelevation of outer rail, centrifugal tendency, effect of coning, weight on each wheel, wheel base, and gauge of rails, whether tight or slack: and it is noticeable that as applied to a case recently reported upon by the officers of the Board of Trade, the theory seems to apply very satisfactorily. It is useless to say that derailment is very uncommon, for in the annual Board of Trade report just issued one line alone records fourteen cases of derailment of passenger engines or vehicles, and another ten, both being lines on which flanges with considerable inclination are used. Altogether, a large number of cases of engines and vehicles leaving the rails is reported, and the comparative frequency with which engines do get off the road inevitably suggests that narrow escapes from derailment with some kinds of engines must be uncomfortably numerous. Experience leads us to feel that fixed wheels on parallel axles answer, on the whole, very well, but it must be admitted that were it not for the effect of custom, and were the subject approached anew with unfettered ideas, any engineer would be inclined to say that either loose wheels must be employed as they are on traction engines, or radiating axles must be used. The latter affords the best solution of the difficulty, for there is no doubt great advantage in fixing wheels to the axles. The paper referred to shows the value of this system, and points to the great advantage derivable from the system of lubricating the flanges of leading wheels, as now largely done on the Continent.

THE GEOMETRY OF SPACE.

By RD. RANDOLPH, C. E.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

THE word geometry is here used, notwithstanding its derivation which seems to limit the science to the earth, because there is no other word whose employment has made it the exponent of the same idea. If Euclid had anticipated the discussions that have lately arisen among mathematicians concerning his work, he would probably have called his science *pandiametry*, and have maintained that it was equally true if space alone existed. He would have rejected the appellation *planimetry*, lately given to it, as false, and insisted that the plane was one of his figures in space, and that when a discussion was based upon the assumption that lines and points were confined to the same plane, it was only for the convenience of description; their coincidence with the plane being only the expression of one of their conditions in space. It will be the endeavor of this article to do what Euclid himself would have done if he could have apprehended the consequences of certain defects and incompleteness in some of the foundation stones of his noble edifice which has nevertheless stood unimpaired for more than two thousand years, but which has of late years given birth among eminent mathematicians to a set of ideas concerning space which strike the ordinary mind as monstrous absurdities.

In VAN NOSTRAND'S MAGAZINE for December, 1876, is a discussion of this subject by Professor Helmholtz, and in the *Popular Science Monthly* for August, 1880, is an article by George B. Halstead, A.M., Ph. D., giving an account of the origin, development, and literature of this new departure in geometry. These seem to be the only sources of information of a popular character that have appeared in this country by which we can learn what the new ideas are. Taking these articles as fair exponents, it seems

that the impossibility of demonstrating some of the propositions upon which geometry is based, has necessitated the existence of modes or species of space, where the truth of the propositions may become apparent. And as the straight line, the square and the cube of geometry correspond to the 1st, 2d and 3d power of arithmetic, which series of powers can be continued without limit, while the series of geometrical magnitudes terminate at the cube, then such abrubt termination must be caused by reaching the limit of the space to which they belong, and that the process may be continued by passing over that boundary line and entering the domain of another space where a new geometrical figure will be formed, corresponding to the 4th power of arithmetic.

The geometrical magnitudes are conceived to be generated in two ways—by multiplication and by motion. But it is not possible to multiply a line by a line, any more than it is to multiply a volume by a volume, a surface by a surface, a pound by a pound, or an hour by an hour. A square yard is not obtained by multiplying a lineal yard by a lineal yard, but by multiplying a square foot by the square of 3. Nor is a cubic yard the 3d power of a lineal yard, but it is the pro duct of a cubic foot by the 3d power of 3. So a lineal yard is the product of a lineal foot by 3. There is nothing in the nature of common space to prevent these multiplications being carried on without limit; the cubic foot or any other element by volume can be multiplied by any power of any number.

It is customary to say that a line is described by the motion of a point, a plane by the motion of a line, and a volume by the motion of a plane. These descriptions would be practically illustrated by saying that a line is the aggregate of the particles of chalk left in the track of a moving piece over a board; a plane is the aggregate of such particles left in the track of a number of contiguous pieces moving together over adjoining paths, and that the volume would be the cloud of particles formed in the air if the board were reversed and they fell to the ground. Now if these particles were only mental concepts of positions divested of all idea of extensions but unlimited in number, we would have the mathematical idea of geometrical magnitudes. But that there is no geometrical term to denote the figure described by the movement of a volume is according to Helmholtz, because there can be no mathematical idea of such a figure distinct from that of a volume, unless we appeal to another mode or species of space where the new figure will become apparent. Taking both the practical and the mathematical view of these magnitudes, it is evident that they are all nothing more than aggregates of points or positions arranged according to certain conditions. A line is an aggregate of points, a plane is an aggregate of lines, and a volume is an aggregate of planes and consequently of points, while all the points are positions in space. None of them are spaces nor parts of space. Therefore, none of the geometrical magnitudes are spaces, parts of space, or modes of space.

But the objection that the movement of a volume results only in describing a repetition of itself is not confined to the volume, for if the straight line be moved in the direction of its length, or if a circular curve be moved about an axis at the center of its circle, a line and nothing but a line will be described. So if a plane be moved in the direction of any of its lines at any point in the movement, a plane and nothing but a plane will be described. If these facts do not occasion the want of a new space, why should the fact that a volume moved in the direction of any of its lines will describe only another volume, require the existence of another mode of space?

It is quite as admissible to conceive a plane as being described by the movement of a point, as by the movement of a line, an idea that may be represented by the movement of the shuttle in disposing of the woof of a fabric. And suppose the line represented by the single thread of the woof, disposed in the form of a plane, to move, then there would be a volume described by a line. And even the point may describe the volume by moving over all the lines of all the planes.

But the principal ground of justification for the new ideas seems to be, that it has been found impossible to demonstrate the truth of certain propositions which lie at the base of Euclidian geometry, and that the difficulties can only be overcome by substituting the method of

Euclid by that of modern algebraical analysis, an instrument which gives access to the new variety of space transcending that contemplated by Euclid. As an illustration of the possibility of a transcendental intellect comprehending a distinct science of geometry belonging to such a space, attention is called to the fact that our intellect *does* comprehend a geometry which belongs to a species of space transcending that of a hypothetical being whose existence is limited to what we know as a mathematical surface. Here it is evident that the three geometrical magnitudes, in which ordinary minds can perceive nothing but variously disposed aggregates of points in space, are assumed to be three varieties of space. And all the properties of lines and angles drawn upon a variety of surfaces as properties peculiar to such distinct kinds of space, whereas the ordinary intellect of Euclid would have considered them as ordinary geometrical figures and magnitudes determined in accordance with prescribed conditions, such conditions being secured by confining them to surfaces of a certain character, themselves being figures in ordinary space.

The writers on this subject find themselves between the horns of a dilemma in being forced to choose between two words, these are the shortest in the English language, but have the most important signification. Some of these writers, Helmholtz for one, place the hypothetical beings *on* the surface, while others place them *in* the surface. If they are on the surface, their existence must be in that space outside of which they are supposed to know nothing. If they are in the surface, then it must have thickness enough to admit them and the *volume* of their persons. The supposition of reasoning beings existing within a geometrical surface, is both logically and mathematically impossible. It would, however, have been perfectly logical to suppose Professor Hilgard and his assistants of the Coast Survey having become monomaniacs owing to their long practice of measuring shortest lines between two points on the great circle of the earth, would utterly deny the proposition that there could not be two shortest lines between two points. And that they could no more have an idea of a shorter line, dipping into the bowels of the earth, between the same points, "than men born blind could have of colors," any such propositions from outsiders meeting with the same contempt as that with which the suggestion of *Alice in the Wonderland*, or *Alice through the Looking-glass*, were received by the inhabitants of those spaces, the dimensions of which the author of those books does not inform us.

But it cannot be denied that from Euclid to Legendre, the Alpha and Omega of the science of geometry, some of the main propositions have not been demonstrated, and some of the definitions have been incomplete and insufficient, while others have assumed as truth that which needed demonstration.

The object of this article is to amend these defects.

The first nine axioms of Euclid (Todhunter's edition) and of Legendre, are the only ones admissible as such. A better name, however, for these statements would have been *tautologisms*, as they are nothing more than repetitions of the same idea in different words. For instance, a whole is equal to all its parts. The meaning of whole is, all the parts, and the expression is equivalent to saying that the whole is equal to the whole. The whole is greater than any of the parts. The meaning of the word part is something less than the whole, so the expression is equivalent to saying that the greater is greater than the less. Things that are equal to the same things are equal to each other; which is, if A has the quantity C, and B has the quantity C, then both A and B have the quantity C, the same quantity. If equals be subtracted from equals the remainders are equal; which is, if A and B are equal to A and B then A is equal to A without B, etc., etc.,

The 11th axiom of Euclid, being the 10th of Legendre, viz., all right angles are equal, should have been expressed in the definition of right angles, viz. Equal angles, each one half of all the angles in a plane formed between a straight line and its extension beyond the vertex of the angles.

The 10th of Euclid, equivalent to the 11th of Legendre, viz., only one straight line can be drawn between two points, is a proposition to be proved; for the definition of a straight line, viz., a

line that does not change its direction at any point, does not justify the assumption, as there is no definition of a direction. A ship which sails constantly in the same direction sails in a circle: and it would not do to say that a direction was a straight line, for that would be reasoning in a circle.

The 12th of Legendre, Euclid having no corresponding one, viz.: the shortest distance between two points is measured on the straight line which joins them, depends upon the truth of the proposition that there can be but one straight line between two points which is not demonstrated.

The 13th of Legendre, Euclid having none to correspond, viz.: through the same point only one line can be drawn parallel to a given line, is another proposition requiring proof, for there is nothing in the definition of a straight line or of parallels by which the points of the lines can be located.

The 12th of Euclid is classed as an axiom by his translator, but by Euclid himself it offered as a *request* that its truth be granted. But in Legendre it is the 21st proposition of the 1st book, where it is pretended to be demonstrated; viz.: If the two included angles formed by two straight lines with a third line are together less than two right angles, the two lines will meet if sufficiently extended, it being understood that all the lines are in the same plane. Starting from the improved proposition that there can be but one straight line between two points, Legendre proves that whenever two straight lines form, with a third line included, angles whose sum is two right angles, they never meet; but does not pretend to prove the converse, that all lines which do not meet form included angles equal to two right angles. For all that is proved to the contrary, they may also not meet with angles less than two right angles. But by applying the term parallel to all lines that do not meet this omission is lost sight of in the conclusion. The demonstration is briefly thus: All lines that do not meet are parallel. Two lines which form included angles equal to two right angles cannot meet. (Supposed to be proved.) Two lines which form included angles less than two right angles must meet, because if they did not they would be parallel. But as the word parallel is only a substitute for the sentence, which do not meet, the conclusion is nothing more than this. They must meet, because if they do not meet they will be lines which do not meet.

As for the postulates, they may all be dispensed with by the proper definition of a point, viz., a position anywhere in space, devoid of extension, but unlimited in number within any limits. A straight line may extend between any two points, means that space will afford all the points of a straight line or any other line, surface or volume between any two points or any number of points. Space will also afford a central point of a line of the arc of an angle, bisecting the line or angle, or another point trisecting them, etc. And as there can be a point on the line representing every possible length less than the whole, there will be one coinciding with any lesser line in space, etc., etc.

Among the definitions there are only two that it is necessary to notice, that of the plane and of the straight line. These definitions are the same with both Euclid and Legendre. That of a plane, viz.: a surface such that if any two of its points are joined by a straight line, every point in the line will be in the plane, is a proposition without proof. Suppose a certain surface should be defined as one, such that if any two points were joined by an ellipse, every point of the ellipse would be in the surface. Such a surface could not exist in space. The definition of a plane cannot be pronounced impossible, but the possibility of it may legitimately be doubted. If such definitions were admissible, the gordion knot of the proposition of the parallels could have been cut long ago by defining parallel lines to be such, that a third line could intersect them at right angles to both at any point of either. But the possibility of this would have properly been doubted and the proof required.

The definition of the straight line is the fundamental defect in our system of geometery, and has caused most of the difficulties connected with it. This definition is the same with both Euclid and Legendre, viz.: a line uniform in its direction at every point. But how can the positions of the points of the line be determined by this definition. Suppose a

circle has been defined as a line having a uniform curvature at every point without defining curvature. But the definition of a circle is perfect when it is upon the condition of all the points being in the same plane. The straight line requires a definition by which all its points may be located quite as imperatively as that of a circle or a sphere: and as there is such a definition as simple and as undeniable as that of a circle, it should be used in any geometry.

The following definitions, propositions, and demonstrations, embracing all the objections to the system of Euclid, are submitted as amendments. It will be seen that all the axioms except the tautologisms are discarded, and all the propositions demonstrated. The discussion has reference to space, pure and simple, and the plane is only used as a convenient mode of expressing conditions in space.

Definitions.

1. Space is universal distance, co-extensive with that which is and that which is not. It is absolutely devoid of conditions, modes or form, but is itself the condition precedent of all existence or action either physical or mental.

2. A point is a position anywhere in space. It is devoid of extension, but unlimited in number within any limits.

3. A distance is the spatial relation of two points. It is a quantity independent of all other points.

4. A line is a succession of points, the length of which is the sum of the distances between its points.

5. A spherical surface is the aggregate of all the points that are equally distant from the same point, called the center of the sphere, the sphere being the aggregate of all such surfaces, having the same center within any given one.

6. A circle is a line, all of whose points are equally distant from the same two points in space.

7. A plane is the aggregate of all the circles which respectively have all their points equally distant from the same two points.

8. A straight line is a line of which all the points are respectively equally distant from the same three points in space.

9. Equal straight lines are those which have the same distance between their terminal points.

10. A straight line is continued when all the points of the continuation, as well as of the line are respectively equally distant from the same three points.

11. A radius is a straight line extending from any point of the circle to any point in space that is equally distant from all the points in the circle. When two such points are at the same distance from the circle each one is the vertex of a cone, and the radius is the cone radius of the circle (called slant height of the cone). The straight line whose points are respectively equally distant from the points of the circle, is the axis of the circular plane and of all the cones which have that circle for base, the cone being the aggregate of all its radii. When the point on the axis is equally distant between the vertices of two cones on the same circle and of equal radii, it is the center of the circle, and the radius to this center is the plane radius of the circle. Two plane radii, one of which is the continuation of the other, form the diameter of the circle.

12. An angle is the extent of the divergence from a common point of two straight lines. When these lines form radii to the same circle the angles are in proportion to the length of that portion of the circle included between them.

13. A triangle is the figure formed in space by the lines including an angle being connected by another straight line between any point on either line.

14. Equal angles are those which have the sides of the triangles to which they belong respectively, equal to each other. (Evident by superposition.)

15. Right angles are equal angles, each being equal to one-half of all the angles formed in a plane between a straight line and its continuation beyond the vertex of the angles. When one line forms right angles with another, it is said to be perpendicular to it.

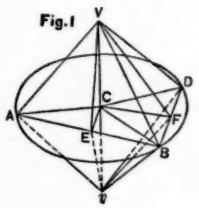
16. Parallel lines are lines in a plane having every point of one at the same distance from the nearest point of the other.

Propositions.

1. Any three points determine the center of the circle to which they belong and all the points of its axis.

Let A, B and D be any three points in space, and let them be connected by straight lines. From the centers, E and F, of these lines, let perpendiculars to them, EV and FV, be extended so as to intersect each other at any point V. Then bring the terminal points of perpendiculars of the same length, E*v* and F*v*, to a common point *v*. Also let two other perpendiculars intersect at a point C equidistant between V and *v*.

Fig. 1



As the angles AEV and BEV are equal, and the sides AE and EB also equal, with the side E V in common, the sides AV and BV are equal (Def. 14) For the same reasons the sides BV and DV are equal. Therefore the three cone radii AV, BV, and DV are equal, which are equal to their counterparts A*v*, B*v*, and D*v* by construction. In the same way the radii AC, BC and DC are proved to be equal. And as C is equidistant between V and *v*, they are radii of the circle ABD (Def. 11). These three points VC and *v* being respectively equidistant from the same three points A, B and D are points of the same straight line (Def. 8). As V and *v* are any points in space an unlimited number of such points may be determined by an unlimited number of intersections of the perpendiculars to the lines AB and BD. This straight line is the axis of the circle ABD, and of all the cones having it for base (Def. 11).

Corollary.—All the plane radii of the circle are at right angles with its axis. For the sides VB and VC are respectively equal to the sides *v*B and *v*C, and the side CB is in common. Therefore, the angle VCB and *v*CB are equal (Def. 14), and are right angles (Def. 15). In the same way the axis can be proved to be at right angles with any radius to the circle.

2. All of the following propositions are demonstrated by a reference to the definition of equal angles (Def. 14).

A triangle with three equal sides has its three angles equal.

A triangle with two equal sides has the angles opposite the equal sides equal.

A triangle with two equal angles has the sides opposite to the equal angles equal.

If in two triangles the sides are respectively equal, the angles opposite to the equal sides will be equal.

If in two triangles two sides are respectively equal, and the angle included between them equal, the triangles are equal in all their parts.

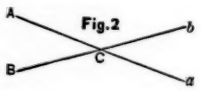
If in two triangles two of the sides are respectively equal, and an angle in each opposite to one of the sides equal, the triangles are equal in all their parts.

If in two triangles two of the angles are respectively equal, and the side included between them equal, the triangles are equal in all their parts.

If in two triangles two of the angles are respectively equal, and a side in each opposite to one of the angles equal, the triangle will be equal in all its parts.

3. The angle formed by two straight lines is equal to that formed by the extension of those lines beyond the common point. The angles AC*b* and A*c*B

Fig. 2



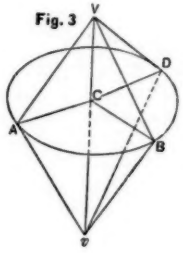
are equal to the angles A*cb* and *a*C*b*, as both sums are equal to two right angles (Def. 15), therefore A*c*B is equal to *a*C*b* (axiom).

4. There can be but one straight line between the same two points.

Let V and *v* be the two points, and let them be the vertices of two cones based on the circle ABD, whose plane is intersected by the two straight lines between V and *v* at any point where they are supposed to deviate from each other. As each point in the same straight line is equidistant from the same three points (Def. 8.), and two of these points, V and *v*, are each equidistant from the points A, B and D, by construction all the

points in either straight line between V and v must each be equidistant from these points. Therefore, neither can intersect the plane of the circle at any

Fig. 3

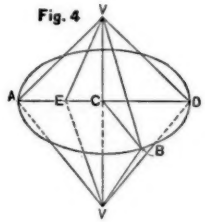


other point than at c, the center of the circle. Therefore, the supposed deviation is impossible; and, as this reasoning applies to any point of supposed deviation, the two lines cannot deviate at any point, and are, therefore, but one line.

5. There can be but one perpendicular to the same straight line extended from the same point.

Let V be the point from which the perpendiculars are to be extended to the straight line ACD, which is the diameter of the circle ABD. Let c, the center of the circle, be the point where one of the perpendiculars intersects the diameter, and let AV, BV and DV be equal. The

Fig. 4



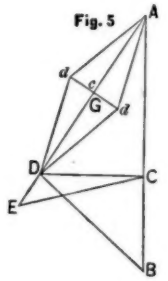
line VC is perpendicular to the diameter, because AV and Ac are equal to DV and Dc, with CV in common (Prop. 2). If there is another perpendicular to the same line it will intersect it on either side of C, at E. But it cannot form equal angles with it, because AE will be less than ED (Prop. 2). Therefore, there can be but one perpendicular from the same point.

Corollary.—There can be but one perpendicular to a plane from the same point; for the same reasoning can be applied to any diameter of the circle ABD.

6. The length of a straight line is less than that of any other line between the same points.

Let ACB be the straight line between the points A and B, and let D be a point in the line not straight which extends between the same points, and let DC be perpendicular to ACB. Then the straight line AD will be longer than AC, and the straight line DB will be longer than CB. For if AD and AC were equal the angles ADC and ACD would be equal (Prop. 2), and both would be right angles, as the angle ACD is so by construction. But this would be two perpendiculars

Fig. 5



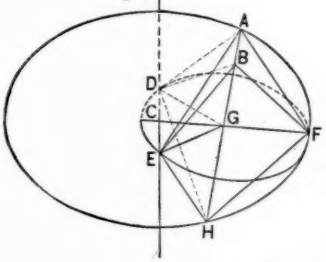
from the same point, which is impossible (Prop. 5). Therefore the lines are not equal. If AD is shorter than AC, let it be extended to E, where it will be equal to AC and connect E and C with a straight line. Then the angles AEC and ACE will be equal, as they are opposite equal sides (Prop. 2). But the angle ACE exceeds the right angle ACD by the angle DCE; therefore, both angles are greater than a right angle, which is impossible, as the sum of any two angles of a triangle is less than two right angles. (Euclid, Prop. 17, Book I.) Therefore, the line AD is neither equal nor less than the line AC, and must be greater. In the same way DB is proved to be longer than CB.

Again, let d be another point of the line not straight on either side of AD, and let a perpendicular extend from d to c on the line AD. Then by the preceding reasoning the straight lines Ad and

*d*D are proved to be longer than A*c* and *c*D. Thus every pair of straight lines is proved to be longer than the single straight line joining the same points. In the same way the line not straight may be continually subdivided, and every pair of straight lines proved to be longer than the single one between the same points. Thus a constantly increasing sum of straight lines is obtained until it reaches the last series of straight lines which are the distances between the ultimate points of the line not straight, if it should be a curve. Therefore, it is longer than the first straight line between the two terminal points.

7. If two points of a straight line are in a plane, all the points of that line are in the same plane.

Fig. 6

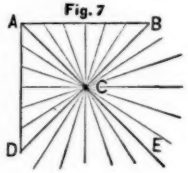


Let A and B be any two points, in the plane of which DCE is the axis and C the center, the points D and E being any points equidistant from C. As a plane is the aggregate of all the circles which respectively have all their points equidistant from the same two points (Def. 7), and as these points are anywhere on the axis at equal distances from the center (Def. 11), the points A and B are respectively equidistant from D and E. Now, let the distance of these points to A and that to B be extended to a common point in the plane, F. Then A and B will be respectively equidistant from the same three points D, E and F. Therefore, all the other points of that straight line are respectively equidistant from D, E and F. (Def. 8.) And as every point in the plane is respectively equidistant from D and E, all the points of the straight line ABGH, whose points are respectively equidistant from D, E and F, are in the same plane.

Corollary.—All the straight lines extending from a common point and intersecting another straight line have a common perpendicular at the common point. For if the intersected line is sufficiently continued, two of the points of intersection are equally distant from the common point. In whatever circle these points may be they will be in the plane to which the circle belongs. And as all its other points are in the same plane the line will be intersected by all the plane radii of the circle, and which has its axis perpendicular to them at the center of the circle.

8. A straight line may be perpendicular to a plane at any point, that is, be at right angles with all the straight lines of the plane which radiate from that point.

Fig. 7



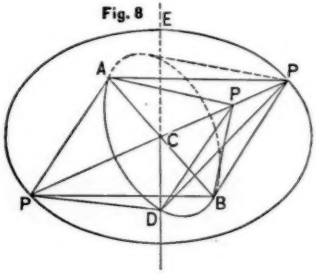
As the axis whose points determine a plane is perpendicular to all the plane radii of its circles (Prop. 1, Corollary); and as a straight line which is intersected by two of these radii has all its other points in the same plane (Prop. 7), it follows that all the straight lines radiating from a common point and which intersect another straight line may have a common perpendicular at that point. If C be any point in a plane and the straight line AB in the same plane, then a straight line can be perpendicular at C to all the straight lines which may radiate from that point and intersect the line AB and consequently to all their extensions between D and E. (Def. 15). So all the lines which radiate from C and intersect the line AD, also in the same planes, and all their extensions between B and E, may have a common perpendicular at C. But as these two sets of lines have AE BDi n common, all the lines which radiate from *c* in the same plane have a common perpendicular at C.

9. If a circle has three of its points in a plane, then all of its points and its center are in the same plane.

As it requires three points to determine the points of the axis of the circle to which they belong; and as the lines which extend from these points to a common point in the axis and form right angles with it are plane radii of the circle, and the common point is the center of the circle (Prop. 1, Corrolary), it follows that the perpendicular to the plane at a point equidistant from the three points of the circle in the plane is the axis of the circle containing those points, and the equidistant point is the center of the circle. Because all the straight lines of a plane which radiate from any point have a common perpendicular at that point (Prop. 8). And as all the plane radii of the circle are at right angles with the axis, all the points of the circle are in the plane.

10. If the line is in a plane, it is a straight line when all its points are respectively equidistant from the same two points in the plane.

Fig. 8



For if the two points, A and B, be connected with a straight line and at a point on this line, C, that is equidistant from A ane B, there be a perpendicular to the plane, ECD, and upon this perpendicular there be a point D, the same distance from C as A and B; then any other points in the plane which are equidistant from A and B will also be equidistant from D on the perpendicular. Let P, P and P be such points, respectively equidistant from A and B. Then if AP and BP are equal and AC and BC are equal, a line from C to P must be at right angles to AB (Def. 14), and the angle DCP is a right angle because the perpendicular DC is at right angles with every line in the plane radiating from C (Prop. 8). The triangles ACP, BCP and DCP having two sides of one respectively equal to two sides of the other, and the included angles equal, the other sides, AP, BP and DP are equal (Prop. 2). Each point being the same distance from the two points in the plane must be the same distance also from the point on the perpendicular. Therefore the points in the line of the plane which are respectively equally distant from the same two points in the plane are respectively equidistant from the same three points in space; which is the definition of a straight line.

11. If the line is in a plane, it is a circle when all its points are equally distant from the same point in the plane.

A circle being a line whose points are all equally distant from the same two points in space, if these two points are connected with a straight line and from a point on this line, equally distant from the other two, straight lines be extended to every point in the circle, they will be at right angles with the first line. (Def. 14). As all the lines in the plane which radiate from any point have a common perpendicular at that point, if the points in the line of the plane are equidistant from this point they are equally distant from the same two points in space, and which are on the perpendicular.

12. If, in a plane, a connected series of equal straight lines from equal angles with each other, those less than two right angles being on the same side of the lines; and these angles are bisected by straight lines, then points on these bisecting lines which are equally distant from the vertex of the angles are the connecting points of another series of equal straight lines forming equal angles with each other, those less than two right angles being on the same side of the lines as of the first series.

Let the first series be composed of the equal lines AB, BC, CD and DE, and connect each alternate point with the lines AC, BD and CE which will be equal (Prop. 2). Then the equal lines A*a*, B*b*, C*c*, D*d* and E*e* which bisect the equal angles ABC, BCD and CDE will bisect the lines AC, BC and CE (Prop. 2), and are at right angles with them (Def. 15). Also the angles AB*a*, BA*b*, BC*b*, CB*c*, CD*c*, DC*d*, DE*d* and ED*e* are equal (Prop. 2).

These being taken from the equal angles of the hypothesis there remain the equal angles *b*B*a*, *b*B*c*, *c*C*b*, *c*C*d*, *d*D*c* and *d*D*e*. These angles being included by sides which are respectively equal in their triangles, the remaining sides *ab*, *bc*, *cd* and *de* are equal. Also the angles opposite to the equal sides of these triangles, A*ba*, C*bc*, B*cb*, D*cd*, C*dc* and E*de* are equal. These being subtracted from the right angles A*bx*, B*ax*, B*cx*, C*bx*, C*dx*, D*cx*, D*ex*, and E*dx*, there remain the equal angles formed on each side of the bisecting lines with the second series of equal lines *ab*, *bc*, *cd* and *de*; the angles being less than a right angle, and the lesser angle being the same side as of the first series. In the same way a third series of equal lines with equal angles less than two right angles will connect points on the bisecting lines equally distant from the vertex of these angles; and the process repeated as long as there is any distance between the bisecting lines.

If the points on the bisecting lines do not coincide with any of the lines connecting alternate points of the series they will occur between them and equidistant from the vertex of the angles. Let *f* and *i* be such points and be connected by the line *fghi*, to be repeated between all the bisecting lines. These lines and the angles which they form with the bisecting lines can be proved to be equal to each other in the same way the second series were, by diagonal lines. At *g* and *h* they cross the lines C*d* and D*c* which were proved to be at right angles with the bisecting lines; and therefore form angles with them less than right angles, as there cannot be two perpendiculars from the same point to the same line (Prop. 5): and the lesser angle must be inside the triangle as the sum of any two angles of a triangle is less than two right angles (Euclid, Prop. 17, Book 1): which shows the lesser angle is on the same side as of the other series. Therefore the proposition is true whatever may be the distance from the vertex of the first series less than the distance to the common point of the bisecting lines if they do meet at such point.

If the lines of the second series are on the side of the greater angles, let the first be represented by the lines *ab*, *bc*, *cd* and *dc*. Then let the lines be drawn at right angles with the bisecting lines from the vertex of the angles *a*, *b*, *c* and *d*. These lines will intersect at A, B, C and D. These points are proved to be equidistant from *a*, *b*, *c* and *d* from the triangles B*ba* and B*bc*, &c., &c., having two angles respectively equal and the included sides *ab*,and *bc*, &c.,&c. equal. The lines AB,BC &c., are proved to be equal to each other by the respectively equal sides of triangles which include the right angle BcC, DcC, &c., which proves the angles BC*c*, DC*c*, &c., to be equal and less than a right angle, as there cannot be two perpendiculars from the same point to the line C*c*. This process can be repeated continually until reaching the given points on the bisecting lines: and if such points should not coincide with the intersections A, B, C, D and E, they will occur between two of them, where the truth of the proposition can be proved as before by the line *fghi*.

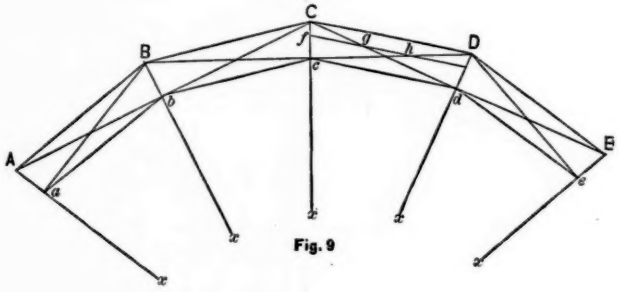


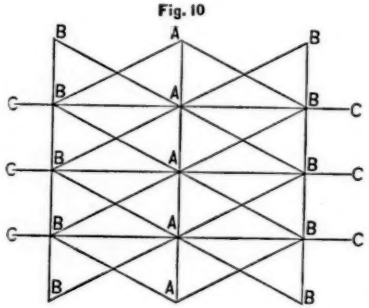
Fig. 9

13. If, in the same plane, several straight lines intersect another straight line at right angles, points on these perpendiculars that are equally distant from

the points of intersection are the same distance apart as the points of intersection.

Let AC be the several straight lines intersecting the straight line AA at the equidistant points A, and let the distances AB be equal. These being at right angles with the line AA, the diagonals AB must be equal (Prop. 2) which would make the triangles AAB equal in all their parts. The equal acute angles AAB being taken

Fig. 10



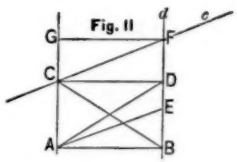
from the right angles AAB leave the angles BAB equal; and the sides including them being respectively equal in the triangles BAB, these triangles are equal in all their parts (Prop. 2). Therefore the lines BB and the angles BBA are equal. If the greater angles BBA are less than right angles, the angles BBC are greater than right angles. If they are greater than right angles the angles BBC are less. As there is a set of these equal lines and angles on each side of the line AA with corresponding angles on the side towards the straight line AA, it follows that if these angles are not right angles the straight line AA is a curved line deflecting in opposite directions at the same points. For, according to the preceeding proposition, if the connected series of straight lines BB form equal angles with each other, which are bisected by the straight lines AB, the points A on these lines which are equidistant from B, the vertex of the angles are the connecting points of another series of straight lines, AA, which also form equal angles with each other less than two right angles, the lesser angles being on the same side of the line AA as of the line BB. Therefore the supposition that the angles BBA and BBC are not right angles is absurd. Therefore they are right angles and the line BB is a straight line. As the greater angles BBA are proved to be right angles and equal to the angles AAB, and as the sides of their triangles, the vertical AB and the diagonal AB, are respectively equal, the triangles are equal in all their parts (Prop 2). Therefore the lines AA are equal to the lines BB, which was the proposition to be proved.

Corollary.—If two straight lines are at right angles to a third straight line, another straight line may be at right angles to both at any point of either.

13. If, in a plane, two straight lines are at right angles to a third straight line, they are parallel; and when they are intersected by a third straight line, the angles formed on the same side of the third line, and, respectively, on the same side of the parallel lines, are equal; and the angles included by the parallels and on opposite sides of the third line are equal.

Let AC and BF be the two straight lines at right angles to the line AB. At any point, C, in the line AC let the line CD be at right angles with AC. Then by the last proposition CD is equal to AB

Fig. 11



and BD is equal to AC; therefore the angle CDB is also a right angle. In the demonstration of proposition 5, it was proved that the shortest line from a point to any point in a straight line was the straight line which was perpendicular to it. Therefore the point A is nearer to the point B than any other point in the line BF, and the point C is nearer to the point D than any other point in the line BF. Likewise the point B is nearer to the point A than any other point in the line AC, and the point D is nearer than any other point in the line AC. Therefore the lines AC and BD are parallel. (Def. 16.) Also

let CG be equal to DF. Then, according to the last proposition, GF is equal to CD and the angles G and F are right angles. Therefore the triangles CFG and CFD are equal in all their parts, for they have two sides respectively equal, one side in common and the included angles G and D right angles. Therefore the angles CFD and that on the other side of the parallel line, *cFd*, are each equal to FCG.

The triangles ADB and CBD being equal in all their parts, the angle ADB is equal to CBD. And the triangles BAD and ABC being equal in all their parts, the angle BAD is equal to ABC. Therefore ABC and CBD are equal to BAD and ADB are equal. Therefore the right angle of the triangle ABD is equal to the other two angles, which proves that the sum of the three angles of a right angled triangle is equal to two right angles. And since every triangle can be divided into right angled triangles by a perpendicular to the longest side from the vertex of the angle opposite; and the sum of the lesser angles of each, being each equal to a right angle, it follows that the sum of the three angles of any triangle is equal to two right angles.

Thus are all of the fundamental propositions of geometry rigorously demonstrated without the aid of doubtful definitions or axioms, and without appealing to a space of x dimensions.

APPROXIMATIVE PHOTOMETRIC MEASUREMENTS OF SUN, MOON, CLOUDY SKY, AND ELECTRIC AND OTHER ARTIFICIAL LIGHTS.*

From "Nature."

Sir William Thomson pointed out that the light and heat perceived in the radiations from hot bodies were but the different modes in which the energy of vibration induced by the heat was conveyed to our consciousness. A hot kettle, red hot iron, incandescent iron, platinum, or carbon, the incandescence in the electric arc, all radiate energy in the same manner, and according as it is perceived through the sense of sight, by its organ the eye, or by the sense of heat,† we speak of it as light or heat. When the period of vibration is longer than one four-hundred-million-millionth of a second the radiation can only be perceived by the sense of heat; when the period of vibration is shorter than one four-hundred-million-millionth of a second, and longer than one eight-hundred-million millionth of a second, the radiation is perceived as light, by the eye.

Pouillet, from a series of experiments, deduced a value of the energy radiated by the sun, equal in British units to about 86 foot-pounds per second per square foot at the earth's surface, or about 1 horse-power to every $6\frac{1}{2}$ square feet of the earth's surface. We may estimate from this the value of the solar radiation at the surface of the sun. The sun is merely an incandescent molten mass losing heat by radiation, and surrounded by an atmosphere of incandescent vapor, so that the radiant energy really comes out from any square foot or square mile of the sun's surface, as from a pit of luminous fluid which we cannot distinguish as either gaseous or liquid. Take, however, instead of the sun, an ideal radiating surface of a solid globe of 440,000 miles radius. The distance of the earth being taken as 93 million miles, the radius of the sun is equal to, say in round numbers, one two-hundredth of the earth's distance, hence the area at the earth's distance corresponding to one square foot of the sun's surface, is equal to 40,000 square feet. The radiation on this surface is (40,000 × 86, or) 3,440,000 foot-pounds, which is therefore the amount of radiation from each square foot of the sun's surface. This amounts to about 7,000 horse-power, which, accord-

* Abstract of lecture at the Glasgow Philosophical Society, by Sir William Thomson, F.R.S.

† Sometimes wrongly called the sense of touch. The true list of the senses, first given, I believe, by Dr. Thos. Reid, makes two of what used to be called the sense of touch, so that, instead of the still too common wrong-reckoning of five senses, we have six, as follows :—

Sense of Force.	Sense of Light.
" Heat.	" Taste.
" Sound.	" Smell.

ing to our brain-wasting British measure, we must divide by 144, if we wish to know the radiation per square inch of the sun's surface, which we thus find to be 50 horse-power.

The normal current through a Swan lamp giving a 20-candle light is equal to 1.4 amperes with a potential of 40 to 45 volts. Hence the activity of the electric working in the filament is 61.6 ampere-volts or Watts (according to Dr. Siemens' happy designation of the name of Watt, to represent the unit of activity constituted by the ampere-volt). To reduce this to horse-power we must divide by 746, and we thus find about 1-12th of a horse-power for the electric activity in a Swan lamp. The filament is 3½ inches long, and .01 of an inch in diameter of circular section; the area of the surface is thus 1-9th of a square inch, and therefore the activity is at the rate of 3 4ths of a horse-power per square inch. Hence the activity of the sun's radiation is about sixty-seven times greater than that of a Swan lamp per equal area, when incandesced to 240 candles per horse-power.

In this country the standard light to which photometric measurements are referred is that obtained from what is known as a standard candle. Latterly, however, objections have been raised against its accuracy. It has been said that differences of as much as 14 per cent. have been found in the intensity of the light given by different standard candles, and that various differences have been observed in the intensity of the light from different parts of the same candle in the course of its burning. The Carcel lamp, the standard in use in France, has been regarded as the only reliable standard. It is, no doubt, very reliable and accurate in its indications, but it should be remembered that its accuracy is greatly owing to the careful method and the laborious precautions taken to secure accuracy. If something akin to the precautions applied to the Carcel lamp by Regnault and Dumas were applied to the production and use of the standard candle, there is little doubt but that sufficient accuracy for most practical purposes could also be obtained with it; probably as good results as are already obtained by the use of the Carcel lamp.

At the Conference on Electrical Units which met in Paris lately, a suggestion was made to use as a standard for photometric measurements the incandescence of melting platinum, and very interesting results and methods in connection with the proposal were presented to the meeting. According to experiments by Mr. Violle, which M. Dumas reported to the Conference, a square centimeter of liquid platinum at the melting temperature gives of yellow light seven, and of violet twelve times the quantities of the same colors given by a Carcel lamp. The apparent area of the Swan filament, being one-ninth of a square inch, is .23 of a square centimeter, and when incandesced to 20 candles must be about as bright as the melted platinum of Mr. Violle's experiment, as the 7 carcels of yellow and 12 of violet must correspond to something like 10 carcels or 85 candles, in the ordinary estimation of illumination by our eyes. The tint of Mr. Violle's glowing platinum cannot be very different from that of the ordinary Swan lamp incandesced to its "20 candles." Thus both, as to tint and brightness it appears that melted platinum at its freezing temperature is nearly the same as a carbon filament in vacuum incandesced to 240 candles per horse-power.

For approximative photometric measurements the most convenient method is certainly that of Rumford, by a comparison of the shadows cast by the sources of light on a white surface. The apparatus necessary are only a piece of white paper, a small cylindrical body such as a pencil, and a means of measuring distances. Ordinary healthy eyes are usually quite consistent in estimating the strength of shadows, even when the shadows examined are of different colors, and with a reasonable amount of care photometric measurements by this method may be obtained within 2 or 3 per cent. of accuracy. The difference in the colors of the shadows is of course due to each shadow being illuminated by the other light.

Arago has compared the luminous intensity of the sun with that of a candle, and estimates it as equal to about 15,000 times that of a candle flame.

Seidel, as Sir W. Thomson had been informed by Helmholtz, estimated the luminous intensity of the moon as about equal to that of grayish basalt or sand-

stone. An experiment on sunlight made in Glasgow on the 8th of this month (since this paper was read), compared with an observation on moonlight, which he made at York during the meeting of the British Association there in 1881, had led him to conclude that the surface of the moon radiates something not enormously different from one-quarter of the light incident upon it. It would be exactly this if the transparency of the Glasgow noon atmosphere of December 8, 1882, had been exactly equal to that of the York midnight atmosphere of September, 1881, referred to below, for the respective altitudes of the sun and moon on the two occasions. The observation on moonlight referred to above showed the moonlight at the time and place of the observation (at York early in September, 1881, about midnight, near the time of full moon) to be equal to that of a candle at a distance of 230 centimeters. The moon's distance (3.8×10^{10} cm.) is 1.65×10^{8} times the distance of the candle. Hence, ignoring for a moment the loss of moonlight in transmission through the earth's atmosphere, we find $(1.65 \times 10^{8})^{2}$, or 27 thousand million million as the number of candles that must be spread over the moon's earthward hemisphere painted black, to send us as much light as we receive from her. Probably about one and a half times as many candles, or say forty thousand million million would be required, because the absorption by the earth's atmosphere may have stopped about one-third of the light from reaching the place where the observation was made. The moon's diameter is 3.5×10^{8} centimeters, and therefore half the area of her surface is 19×10^{16} square centimeters, which is nearly five times forty thousand million million. Thus it appears that if the hemisphere of the moon facing the earth were painted black and covered with candles standing packed in square order touching one another (being say one candle to every five square centimeters of surface), all burning normally, the light received at the earth would be about the same in quantity as estimated by our eyes, as it really is. It would have very much the same tint and general appearance as an ordinary theatrical moon, except that it would be brightest at the rim and continuously less bright from the rim to the center of the circle where the brightness would be least.

The luminous intensity of a cloudy sky he found about 10 A. M. one day in York during the meeting of the British Association to be such that light from it through an aperture of one square inch area was equal to about one candle. The color of its shadow compared with that from a candle was as deep buff yellow to azure blue, the former shadow being illuminated by the candle alone, the latter by the light coming through the inch hole in the window shutter.

The experiment on sunlight of last Friday (December 8) showed, at 1 o'clock on that day, the sunlight reaching his house in the University to be of such brillancy that the amount of it coming through a pin-hole in a piece of paper of .09 of a centimeter diameter produced an illumination equal to that of 126 candles. Tihs is 6.3 times the 20 candle Swan light, of which the apparent area of incandescent surface is .23 of a square centimeter or 3.8 times the area of the pin-hole. Hence the sun's surface as seen through the atmosphere at the time and place of observation was 24 times as bright as the Swan carbon when incandesced to 240 candles per horse-power. By cutting a piece of paper of such shape and size as just to eclipse the flame of the candle and measuring the area of the piece of paper, he found about 2.7 square centimeters as the corresponding area of the flame. This is 420 times the area of the pin-hole, and therefore the intensity of the light from the sun's disc was equal to (126×420) about 53,000 times that of a candle flame. This is more than three times the value found by Arago for the intensity of the light from the sun's disc as compared with that from a candle-flame; so much for a Glasgow Dec. sun!

The .09 cm. diameter of the pin-hole, of the Glasgow observation, subtends at 230 centimeters distance, an angle of 1/2556 of a radian; which is 23.7 times the sun's diameter (1/108 of a radian). But at 230 cm. distance the sunlight through the pin-hole amounted to 126 times the York moonlight (which was 1 candle at 230 cm. distance). Hence the Glasgow sunlight was $[(23.7)^{2} \times 126$ times or] 71,000 times the York moonlight. We cannot therefore be *very far* wrong in estimating the light of full

moon as about one-seventy-thousandth of the sunlight anywhere on the earth. This, however, is a comparison which, because of the probably close agreement of the tints of the two lights, can probably be made with minute accuracy; and we must therefore not be satisfied with so very rough an approximation to the ratio as this 70,000. A lime light, or magnesium light, or electric arc-light, carefully made and remade with very exactly equal brilliance, for each separate observation of sunlight and moonlight, might be used for intermediary.

CURRENT-METER MEASUREMENTS IN THE RHINE, BELOW THE BRIDGE OF CONSTANCE.

By ADAM BAUM.

"Allgemeine Bauzeitung." Abstracts of the Institution of Civil Engineers.

In studies for lowering the high-water level of the Boden See, it became necessary to ascertain the discharge with different levels of the water in the lake. The measurements were made at a cross-section of the Rhine, below the bridge of Constance, near the point of discharge from the Boden See. For ordinary conditions of the water-level the stream has here a conveniently bounded profile. At the highest water level the stream floods the banks, but the quantity flowing beyond the limits of the channel is vanishingly small compared with that flowing in the ordinary bed. The cross-section has a breadth of 136.13 meters (450 feet), and a maximum depth of 11.4 meters (37½ ft.)

Surface-fall.—To determine this, nine gauges were fixed, six on the left shore and three on the right. The highest on the left, the Rhein-thorthurm gauge, was that to which all measurements were referred. To determine the surface-fall with different levels of the water in the lake, thirty-three sets of readings of the levels on the gauges are available.

The author has plotted these results, which exhibit great differences. A discussion of them leads to the formula—

$$J = 0.000067541 - 0.00000173474u,$$

where J is the relative fall, and u the height of the water-surface on the principal gauge. The zero point on this gauge is at the highest known water-level.

Measuring Apparatus.—The supports for the current-meter were placed on a platform between two coupled boats, each 33 feet long by 6 feet beam. The current-meter was attached to a fixed vertical T-iron (4 inches by 2⅜ inches). The arrangements for fixing this and for raising and lowering the meter are described. The current-meter was fixed with its axis normal to the cross-section, and was not directed by a rudder or vane. It had a screw of 4.7 inches diameter driving a worm-wheel, making one rotation to one hundred of the screw. The worm-wheel carried a pin making electrical contact once in each revolution, so that a bell sounded at each hundred rotations of the screw.

Determination of the Constants of the Meter.—The observations give merely the time of one hundred revolutions, and the velocity of the water is not directly proportional to the speed of the meter. It is necessary, therefore, to find a relation between the time z per hundred revolutions, or the number of revolutions n per second, and the velocity v of the water. Then $n = \frac{100}{z}$, and $v = f(n)$ is the relation required. To determine this relation a number of trials were made in still water. The boats carrying the meter were guided by a fixed wire rope, and moved by a windlass. One observer noted the time of one hundred revolutions, and another the distance traversed along the guide rope. Applying the method of least squares, the author finds the equations—

(α) In the direction of the wind,
$$v = -0.1271 + 0.3706n.$$

(β) Against the wind,
$$v = 0.12779 + 0.25104n.$$

(γ) From both combined,
$$v = 0.02514 + 0.259533n,$$

v being in meters per second. The discrepancy of these equations, which is hardly explainable as due to the wind, which was always very slight, leads the author to discard them. The negative value of one of the constants is also impossible. The proceeding ultimately adopted was this: The rotations per second and corresponding velocities were plotted in a diagram as abscisses and ordinates. Through the points so found a provisional straight line was drawn. One point in this line near its upper end was assumed as accurately fixed, and from this and the $m-1$ other values of n and v equations were formed to determine the two constants. The arithmetical mean of these two values gave the following equations:

(α) In the direction of the wind,
$$v=0.06037+0.28168n.$$

(β) Against the wind,
$$v=0.08844+0.26892n.$$

(γ) Mean of both,
$$v=0.0744\ +0.2753n.$$

The difference in the first constant is narrow, and indicates a small current reverse to the direction of the wind, at the depth at which the meter was placed. The author discusses some results with the meter differently supported, and concludes that formula (γ) may be used in reducing the observations with the meter in all cases.

Surface-velocity Curves.—The author gives the surface-velocity curves for three conditions of the river. The curves are too irregular to be approximated to any geometrical figure.

Vertical-velocity Curves.—These in general are similar to those obtained in other researches. The exceptional form of some of the curves is due (1) to the friction of the contact-maker, which, when the velocity was very small, had a proportionately greater effect in retarding the meter; (2), to peculiarities of the river-bed, the irregularities of which influenced the position of the point of greatest velocity; (3), to the nearness of the point of discharge from the Boden See. The velocity at each point was due partly to the surface fall at the section, partly to the pressure of the Boden See considered as a reservoir. The peculiarly deep position of the point of greatest velocity in some of the curves may be explained as due to these causes.

Variations of the Velocity at one Point.—The meter was fixed in a selected position in the cross-section, and from the bell signal the time of each successive hundred revolutions was taken for a period of two hours. The results plotted show a continual variation of the velocity. Further, a wavy line can be drawn through the observations, taking a mean position between the shorter oscillations, and having a length from crest to crest corresponding to a period of about an hour.

In order, therefore, to ascertain the accurate velocity at any one place in the cross-section, it would be necessary to extend the observations over the whole of such a period. That is, to observe the time of about six thousand revolutions of the meter. But as this is not practicable, the observations taken in short periods must always be affected by considerable irregularities.

The author then considers observations on a series of verticals, to determine the variation of velocity with varying level of the river. By dividing the area of a vertical velocity-curve by the depth of the river, the mean velocity at that vertical in the given condition of the river is obtained. Setting off these mean velocities as abscisses, with the gauge-reading as ordinate, curves are obtained giving the mean velocity at each vertical for every condition of the river. The curves show great irregularities. These are due to various causes. Partly to the variation of the velocity at each point already discussed, partly to the rising or falling of the water during the observations, partly to the boat not being fixed exactly normal to the cross-section, and partly to the whirling motion of the water, which the author discusses at length.

The discussion of the vertical and horizontal velocity-curves cannot be rendered intelligible without the original drawings.

Discharge.—From the curves of the observations the author obtains a table giving the mean velocity on each vertical for each foot fall of the water-surface of the gauge from 2 feet to 12 feet. The discharge for each of these conditions of the river is obtained by multiplying the area of the cross-section between each

pair of verticals by the mean of the mean velocities on those verticals. For a portion of the section towards each shore the velocity for low conditions of the river was zero. The mean radius and mean velocity has also been reckoned for each of the same gauge readings.

Comparison of measured Discharge with Formulas.—The author has calculated the mean velocity of the river for each level from 2 feet to 12 feet on the gauge by seventeen well-known formulas. The calculated values show very great departures from the measured velocities, especially for low conditions of the river. The author then recalculates from the measured velocities the constants of the formulas, choosing the observations with 5 feet gauge-reading for formulas with one constant, and those at 5 feet and 10 feet for formulas with two constants. Recalculating the mean velocity for all levels of the river with these new constants, there are still extremely wide departures from the measured velocities.

The author then describes some observations with surface-floats in those parts of the river where the velocity was too small for the accurate use of the current meter. Also some observations with rods made to check the observations with the current-meter. The rods were of wood, projecting 20 to 24 inches above water, and reaching to within 16 to 24 inches of the bottom. The agreement with the current-meter observations is satisfactory, but the floats give in general a slightly higher value of the mean velocity than the meter. The difference is due partly to the path of the rods not being quite exactly known, part to the rods not extending to the bottom of the river.

CORRECTIVE DIVISION: AN ABRIDGEMENT OF THE ORDINARY PROCESS OF LONG DIVISION.

By J. BRUEN MILLER, M. S.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

DIVISION, or the process of ascertaining how many times one number is contained in another, is universally performed by one method. For convenience, the terms "short" and "long" division have been adopted, but in reality the two processes are identical. Short division is the term employed to signify the division of one number by another, when the divisor is a single digit, or when it is one of the numbers whose multiples from 1 to 12 inclusive are obtained from the multiplication table. In short division, the progressive steps of multiplication, subtraction and addition, which are essential to division, are performed *mentally*, familiarity with the multiplication table enabling the student to reach the desired ends without the necessity of ciphering the intermediate results. In long division, the divisor is so large a number that the progressive steps cannot be performed mentally, and the student is compelled to have recourse to ciphering. In method and analysis the processes are identical. Short division is *mental* long division, and long division is *written* short division.

It is true that examples which should properly be solved by long division are often solved by short division, owing either to the readiness of reckoning possessed by the student, or to the peculiar nature of the divisor, as for instance, if the latter be a multiple of 10, or a factor of 100, etc. Instances of this sort are numerous, but no matter how great the skill and reckoning possessed by the student or operator, they are the exceptions, not the rule; and as the divisor increases in denomination, there is a decrease in these instances, and long division offers the only method of solution. The use of logarithms cannot properly be called a method of division, since it requires a series of previously computed tables, and is at best an approximation.

In the new process of "corrective division," which is herein treated, the necessity of performing at length the various progressive steps essential to

division, and of checking and proving the work by writing each successive result is obviated in performing the task by short division, with *mental corrections* at each and every step. For this reason the process is called *corrective* division.

Its discovery was due to an investigation of the properties of numbers with a view to abridging the labors of multiplication and division, and while simple in detail, it was not reached until weeks of labor had been expended in other directions.

The process of "corrective division" is a complete analysis, purely arithmetical, requiring no algebraic formulæ to demonstrate its unvarying accuracy, while to comprehend the method, and to analyze its various steps, no greater intelligence is needed than to acquire a thorough understanding of short and long division.

The new and the old methods are contrasted in the following examples:

Let it be required to divide any number as 847125 by any other number as 67. Without entering into an analysis of the well-known process of long division, it is sufficient to perform the task which appears with its progressive steps as follows:

```
67)847125(12643.656+
   67
  ---
   177
   134
  ---
    431
    402
  ---
     292
     268
  ----
      245
      201
     ---
       440
       402
      ---
        380
        335
       ----
         450
         402
```

In the above example all the indicated steps must be performed in full, and to obtain the quotient as far as reached, sixty figures are required, with eight trial divisions, eight multiplications, and eight subtractions and subsequent additions. It is true that adepts and accountants may multiply and subtract at the same time, thus saving the labor of writing every subtrahend, but the economy of time is inconsiderable. The above process is almost universally employed in its entirety.

Now let it be required to solve the same problem by corrective division. When completed it will appear as follows:

```
7
6.7 )84712.5
 .3 )12643.656+
```

Analysis: If any number, as 847125 be divided by any other number as 67, the quotient will be equal to the first number divided by 10, and this result again divided by the second number divided by 10 $\therefore$ $847125 \div 67 = 84712.5 \div 6.7$. But dividing 84712.5 by 6.7 is equivalent to dividing 84712.5 by 7 *and to each partial remainder adding the product of the last obtained digit of the quotient and the difference between* 7 *and* 6.7 *which is* .3. Proceeding upon this principle, 7 is contained in 8 once, with a remainder of 1. To this remainder add the product of the last digit obtained in the quotient 1, by the difference between 7 and 6.7 or .3, =1.3. Multiplying 1.3 by 10, =13 and adding the second figure of the dividend 4, =17, the process is continued as before.

$17 \div 7 = 2$ with a remainder of 3;
$[3 + (2 \times .3)] \times 10 + 7 = 43$;
$43 \div 7 = 6$ with a remainder of 1;
$[1 + (6 \times .3)] \times 10 + 1 = 29$;
$29 \div 7 = 4$ with a remainder of 1;
$[1 + (4 \times .3)] \times 10 + 2 = 24$;
$24 \div 7 = 3$ with a remainder of 3;
$[3 + (3 \times .3)] \times 10 + 5 = 44$;
$44 \div 7 = 6$ with a remainder of 2;
$[2 + (6 \times .3)] \times 10 + 0 = 38$.

Here since a cipher was added to the last figure of the dividend, the decimal point must be written were the dividend without decimals, but as the decimal point occurs before the last figure 5, the decimal point must be written in the quotient at a corresponding number of places, $\therefore$ the last obtained figure of the quotient 6, is a decimal. Continuing, $38 \div 7 = 5$ with a remainder of 3; $[3 + (5 \times .3)] \times 10 + 0 = 45$; $45 \div 7 = 6$ with a remainder of 3;

$[3+(6\times.3)]\times10+0=48$, etc. Thus the process may be continued to an indefinite number of places.

If this analysis is not sufficiently clear it may be amplified as follows:

If 7 is contained in 8 once with a remainder of 1, then will 6.7 be contained therein once with a remainder of 1 plus once the difference of 7 and 6.7 or $.3=1.3$, or in the next lower denomination, 13, whence adding it to the units in that lower denomination, 17 is obtained as the next partial dividend, etc.

In using the corrective method of division it is simpler to prefix each prime remainder to the next figure of the dividend and add the product of the last obtained quotient digit and the differential number as before, omitting the decimal. Thus: $8\div7=1$ with a remainder of 1; $14+3=17$; $17\div7=2$ with a remainder of 3; $37+6=43$; $43\div7=6$ with a remainder of 1; $11+18=29$, etc. It is not essential, though advisable, to write either the 7 or the 3, as they may be carried mentally, nor is it necessary to employ decimals except in the dividend and quotient, as in ordinary division.

Thus the labor of division is abridged in the corrective method, by employing but one fourth the number of figures used in long division, and by rapid mental calculation in place of the full written results. Having observed the corrective process as applied when the divisor contains but two digits, the following rules may be deduced:

I. *Above the divisor write the digit next higher than the first digit of the divisor, and point off the right-hand figure of the dividend.*

II. *Below the divisor write the difference between the second digit of the divisor and* 10.

III. *Proceed as in short division, adding to each partial dividend the product of the last obtained digit of the quotient and the digit below the divisor.*

A General Method of Corrective Division.

While the foregoing rules are applicable to any problem where the divisor is below 100, it is necessary to obtain general rules applicable where the divisor contains any number of digits. Before obtaining these rules, a system of terminology will be adopted to simplify the analysis. Thus if any number, as 8432, be a divisor, it is subtracted from the next highest integral multiple of the same denomination, which is 9000, leaving a remainder of 568. For convenience, 9 is called the *base*, since it is the number on which an hypothetical division is based, the number 568, the *whole difference*, and the numbers 5, 6 and 8 the *first*, *second* and *third difference* respectively. The same terms are employed in every case, the foregoing being taken as an example, and the differences are enumerated from left to right.

Let it now be required to divide any number, as 9876465309, by any other number over 100, as 88789:

$$\begin{array}{l} 90000 \\ 88789 \\ 1211 \end{array}\Big)\,987646.5309 \;\overline{}\; 111235.23532+$$

In this example 9 is the *base*, 1211 the *whole difference*, 1 the *first difference*, 2 the *second difference*, 1 the *third difference* and 1 the *fourth difference*.

Proceeding as in the previous case, four places are pointed off equaling in number the four ciphers following the base.

Analysis: If 9 is contained in 9 once, with a remainder of 0, 8.8789 will be contained therein once, with a remainder of 0 + once the whole difference taken as a decimal, *but it is only the product of* 1 *and the first difference which will affect the next partial dividend*, hence, as in the first problem, $08+(1\times1)=9$ which is the second partial dividend; $9\div9=1$ with a remainder of 0. To obtain the third partial dividend, it is necessary not only to add the product of the last obtained quotient digit and the first difference, *but also the product of the previous or last but one quotient digit obtained and the second difference*, since in the multiplication of the whole difference by the first digit of the quotient, the product of this digit and the second difference would have affected this denomination, and consequently the third partial dividend; hence to obtain this partial dividend, 7 is added to $(1\times1)+(1\times2)$, $=10$; $10\div9=1$ with a remainder of 1. To obtain the fourth partial dividend it is necessary not only to add the products of the last obtained quotient digit and the first difference, and the last but one quotient digit and the second difference, *but also the product of the first quotient*

digit or the last but two obtained, and the third difference, since *two* multiplications of the whole difference have previously taken place which will affect the next partial dividend, it being multiplied successively by the first two digits of the quotient, and in thus multiplying, the product of the first quotient digit, or the last but two obtained, by the third difference will affect the fourth digit of the whole dividend, as will also the product of the last but one and the second difference and the product of the last and the third difference; hence the entire sum to be added, or the *correction* as it will be termed, is $(1\times1)+(1\times2)+(1\times1)=4$. Adding this to the remainder 1 prefixed to the fourth figure of the dividend, or 16, which will be called the prime partial dividend, 20 is obtained as the fourth complete partial dividend, $20\div9=2$ with a remainder of 2. To obtain the fifth partial dividend, the prime partial dividend, 24, is added to the products of the *last obtained quotient digit and the first difference, the last but one obtained, and the second difference, the last but two obtained* and the *third difference, and also of the last but three obtained, or the first quotient digit and the fourth or last difference,* since there have been three previous multiplications affecting the fifth digit of the whole dividend and therefore the fifth partial dividend. Hence the fifth partial dividend $=24+(2\times1)+(1\times2)+(1\times1)+(1\times1)=30$; $30\div9=3$ with a remainder of 3. To obtain the sixth partial dividend the sixth prime partial dividend 36 is added to the products of *the last obtained quotient digit and the first difference, the last but one obtained and the second difference, the last but two obtained and the third difference, and the last but three obtained and the fourth difference. The last but four obtained, or the first quotient digit* no longer affects the partial dividends, since its multiplication by the whole difference only affected the *second, third, fourth and fifth digits of the whole dividend, and hence only the second, third, fourth and fifth partial dividends.* Therefore the sixth partial dividend will be $36+(3\times1)+(2\times2)+(1\times1)+(1\times1)=45$; $45\div9=5$, with a remainder of 0; here is inserted the decimal point, and the division may be carried out into decimals in the same manner, remembering that it is only the last four quotient digits obtained that affect the partial remainders. Thus $05+(5\times1)+(3\times2)+(2\times1)+(1\times1)=19=$ seventh partial dividend. $19\div9=2$ with a remainder of 1; continuing and writing the products without the parentheses,

$13+2+10+3+2=30=$eighth par. div.
$30\div9=3$ with a remainder of 3;
$30+3+4+5+3=45=$ninth " "
$45\div9=5$ with a remainder of 0;
$09+5+6+2+5=27=$tenth " "
$27\div9=3$ with a remainder of 0;
$00+3+10+3+2=18=$eleventh " "
$18\div9=2$ with a remainder of 0;
$00+2+6+5+3=16=$twelfth " "
etc.

Hence the process may be carried to an indefinite number of decimal places, and the following rules may be obtained for corrective divsion as applied with any divisor:

I. *Write the divisor and dividend as in short division, and point off from the right as many figures minus one, in the dividend, as there are figures in the divisor.*

II. *Above the divisor place the next highest multiple of the same denomination, and subtract the divisor from it. The factor of the multiple will be the* BASE, *and the remainder the* WHOLE DIFFERENCE.

III. *Divide as in short division, by the base, and to each prime partial dividend add the products of the differences with the last obtained digits of the quotient in inverse order.*

It is unnecessary to state that the ordinary student will find no difficulty in performing the various computations *mentally*, and that no figures need be employed save those of the divisor, dividend, whole difference and the quotient, and the base.

Secondary Corrections.

The above general rules are applicable to any problem in division, but occasions occur when the numbers obtained in the quotient would by this process be in excess of 9, or consist of two digits, which would add to the difficulty of computation. For this reason a system of secondary correction may be employed which will obviate this objection to the method of corrective division. The fol-

lowing example will illustrate this system:

Let it be required to divide any number, as 43571206 by any other number, as 7648:

$$\begin{array}{r} 8000 \\ 7648 \overline{)43571.206} \\ \underline{352} \\ 5686.961803+ \\ 97.07\ 91 \end{array}$$

Following rules I and II as deduced from the preceding problem, the base is found to be 8, and the whole difference 352. Proceeding by rule III, $43 \div 8 = 5$, with a remainder of 3; $35 + 15 = 50$; $50 \div 8 = 6$ with a remainder of 2; $27 + 18 + 25 = 70$; $70 \div 8 = 8$ with a remainder of 6; $61 + 24 + 30 + 10 = 125$. Now were 125 divided by 8, the quotient would be 15, with a remainder of 5, and it would be necessary to consider 15 as a single quotient number in the computation. To obviate the difficulty which the introduction of a number of two digits would involve, it is *presumed* that the previous quotient digit had been computed as *one in excess of what it was written*, or in this case, 9 instead of 8, and the corrected number is written below. Now if 9 had been originally written as the previous quotient digit, or *if the previous quotient digit had been increased by* 1, it is evident that the partial dividend ensuing would have been *decreased by* 80, while to it would have been *added once the first difference;* hence, the *corrected partial dividend* would be the *uncorrected partial dividend increased by the first difference and diminished by ten times the base* or $125 + 3 - 80 = 48$. Presuming that this had been done the work is continued as before; $48 \div 8 = 6$, with a remainder of 0; $02 + 18 + 45 + 12 = 77$; $77 \div 8 = 9$, with a remainder of 5; $50 + 27 + 30 + 18 = 125$. Here it is again seen that the partial dividend contains 8, more than 9 times, and that again would a quotient of more than one digit result, and proceeding as before, 1 is added to the previous quotient digit. But the previous digit was 9, hence it would become 10 by this addition, resulting again in a factor of two digits. To obviate this second difficulty it is presumed that the *next but one quotient digit had been computed as one in excess of what it was written*, or 7 instead of 6. Had this been done originally, there would have been a *decrease of ten times the base* in the ensuing partial dividend, and an *increase of once the first difference* or a *total decrease of* 77, whence the next ensuing partial dividend would have been 0; and the quotient digit 0, whence the second ensuing partial dividend would have been $00 + 0 + 35 + 18 = 53$. But this same corrected partial dividend may be reached by direct means. Thus, as before, the previous quotient figure, 9 is increased by one, which will render it 0, and that next previous, 7. The uncorrected partial dividend, 125, is decreased as before by ten times the base or 80, leaving a remainder of 45, and increased by once the first difference or 3, and also once the second difference, since the last but one quotient digit obtained is increased by one, and hence affects the second ensuing partial dividend; hence the corrected partial dividend is $45 + 3 + 5 = 53$. Proceeding as before, after correcting the quotient, $53 \div 8 = 6$ with a remainder of 5; $56 + 18 + 0 + 14 = 88$. Adding one to the previous quotient figure, making it 7, and subtracting from 88, $80 - 3 = 77$, the next corrected partial dividend is found to be 11; $11 \div 8 = 1$, with a remainder of 3; $30 + 3 + 35 + 0 = 68$; $68 \div 8 = 8$, with a remainder of 4; $40 + 24 + 5 + 14 = 83$; making the correction as before stated, $83 + 3 - 80 = 6$, with the previous quotient figure 9; $6 \div 8 = 0$, with a remainder of 6; $60 + 0 + 45 + 2 = 107$. Proceeding as before in correcting the quotient, the last obtained quotient digit becomes 1; taking the uncorrected partial dividend, 107, from it is subtracted $80 - 3 = 77$, $= 30$; $30 \div 8 = 3$, with a remainder of 6, and the process may be continued indefinitely.

From the foregoing may be deduced the following rules:

I. *If a partial dividend in corrective division contains the base more than* 9 *times, add one to the previous quotient digit, and subtract ten times the base minus the first difference from the partial dividend, thus obtaining a corrected partial dividend with which proceed as before.*

II. *If at the same time the last obtained quotient digit be a* 9, *add both the first and second differences to the partial dividend minus ten times the base; if the last two quotient*

figures be both 9's, add the first, second and third differences; if the last three be 9's add the first, second, third and fourth differences, and, in general, as many differences in order as there are 9's plus one in the previous adjacent quotient digits, until the differences are exhausted or until the last difference be added.

CORRECTIONS BY INSPECTION OR ALLOWANCE.

The labor of secondary correction may be greatly abridged, by *allowing for operations to be performed*, after inspecting the factor. Thus in the previous example, the student may proceed as follows:

$$\begin{array}{r|l} 8000 & \\ 7648 & 43571.206 \\ 352 & \hline & 5697.071913+ \end{array}$$

43÷8=5 with rem. of 3;
35+15=50=second par. div.
50÷8=6 with rem. of 2;
27+18+25=70=third " "
70÷8=9 with rem. of −2;
−19+27+30+10=48=fourth " "
or to amplify the process,
1+27+30+10−20=48= " " "

In this case *allowance* is made for a large partial dividend ensuing, and from this dividend 20 *is subtracted* to add 2 to the previous partial dividend making the latter 72, a multiple of 8. By this means, secondary correction is almost wholly avoided. In general the operator or student may at a glance determine the sum that may safely be allowed as forming the minimum of the succeeding partial dividend, with a view to increasing the present partial dividend to a multiple of the base, and should this sum be overestimated, the necessary re-correction may be made by the *reverse* of the foregoing rules for secondary correction.

TO OBTAIN AN INTEGRAL REMAINDER.

The method of corrective division is especially adapted to problems where a large number of decimal places are required in the quotient, but there are many occasions where the quotient is desired in the form of an integer with a proper fraction, or the remainder is desired as an integer. This result may be obtained as follows:

Let it be required to divide any number as 843217 by any other number as 8679, and to obtain an integral remainder or to express the quotient as an integer with a with a proper fraction.

$$\begin{array}{r|l} 9000 & \\ 8679 & 843.217 \\ 321 & \hline & 97.1354 \\ & \quad 8679 \end{array}$$

Proceeding as before, 84÷9=9 with a remainder of 3; 33+27=60 = second partial dividend; 60÷9=7 with remainder of −3; 2+21+18−30=11; should the process be continued as before, the quotient would contain decimals which it is desired to avoid, hence recourse is had to the following method:

11÷9=0, with a remainder of 11;
111+0+14+9=134; 134÷9=0,
with a remainder of 134;
1347+0+0+7=1354; 1354÷9=0,
with a remainder of 1354;
13540+0+0+0=13540; 13540÷9=0,
with a remainder of 13540.

This last step was unnecessary and was inserted merely to amplify the analysis. It will be seen that with the previous step the differences disappear as factors, hence that 1354 will be the integral remainder. This remainder may be obtained directly as follows:

RULE.—*After obtaining the integral quotient affix to the prime remainder, with its proper sign, as many ciphers as there are decimal places in the whole dividend; and add thereto, the decimal of the dividend, the product of the last obtained quotient digit and the first difference prefixed to the same number of ciphers less one, the product of the last but one quotient digit and the second difference with the same number of ciphers less one, etc., the product of the last obtained quotient digit and the second difference, with two less ciphers affixed, the product of the last but one quotient digit and the third difference with two less ciphers affixed, etc., until the differences disappear as factors, when the total sum is the integral remainder.* In the above example the following are the additions:

−3000=prime remainder with three ciphers affixed.
+ 217=decimal of the dividend.
+2100=product of last quotient digit and first difference, with two ciphers affixed.
+1800=product of last but one quotient digit and second difference, with two ciphers affixed.
+ 140=product of last quotient digit and second difference, with one cipher affixed.
+ 90=product of last but one quotient figure and third difference, with one cipher affixed.
+ 7=product of last quotient figure, with third difference.

1354=integral remainder.

It may be observed in this connection that the process of corrective division applied when the dividend is a multiple of the divisor, or when the divisor is contained in the dividend an exact number of times, will result in one of two quotients. If an integral remainder be sought, the quotient will be the correct quotient less one, with an integral remainder equal to the divisor, when the quotient is corrected by adding to the units and the remainder becomes 0. If the quotient be sought in decimals, there will result a repitend of .99999+, which is equivalent to 1, and which may, as in the preceding case, be added to the units, or the correction may be made at any point according to previously obtained rules.

Suggestions.

Though the foregoing rules will enable the student to solve any problem in division by the corrective method, instances occur where the labor of computation may be abridged. Thus when the first figure of the divisor is 1, 2, 3, or, in general, less than 5, the base will be less than 2, 3, 4, or in general, less than 6, and there is some difficulty experienced in employing the method of correcting by inspection or allowance, as previously explained. For this reason it is often convenient to employ the following rule, bearing in mind that it is not essential to the accuracy of the result.

If the divisor be such a number that it may be multiplied by 1, 2, 3, 4, 5, 6, 7, 8 *or* 9, *without causing its first figure to exceed* 9, *so multiply it, and proceeding with this product as a divisor obtain a quotient by corrective division, and multiply this quotient by the same number by which the divisor was multiplied, to obtain the correct quotient.*

By this means a higher base is obtained and the work simplified. If the first figure of an original or corrected divisor be 9, the base will be 10 and not 1, as might on first inspection be supposed.

It will also be found convenient to *check* the work by writing each prime remainder at the upper right-hand corner of each figure of the dividend; thus if confusion arises, or an error is made in computation, the work may be taken up at any previous point. This method, or rather *check system*, is not essential to accuracy, but is, like the former, one of convenience. It is illustrated thus:

$$\begin{array}{r} {\scriptstyle 900} \\ 843 \\ 57 \end{array}\Big)\,4\,6\,1\,0.\,7\,3 \qquad {\scriptstyle 1\ 0\ 1\ -6-6-4\ 3-3\ 0\ 6}$$

$$5\,4\,6.\,9\ 430604+$$

Explanation: $46 \div 9 = 5$ with a remainder of 1, write 1 at the upper right-hand corner of the second figure of the dividend; $11+25=36$; $36\div9=4$ with a remainder of 0; write 0 as before above the third figure of the dividend, and proceed giving remainders the minus sign, if they be minus, and thus insure a check upon the work.

Corrective Division by Subtraction.

The same process of corrective division may be employed by correcting the prime partial dividends by *subtractions*, when the foregoing rules may be *inversely applied*. Thus, let it be required to divide any number as 472164 by any other number as 913. Write the divisor and dividend as in short division:

$$913\overline{)4721.64}$$
$$517.156+$$

In this case 9 may by analogy be termed the *base*, and 13 the *whole difference*, and the division may be performed as before, *subtracting* the products of the differences and the quotient digits instead of *adding* them, thus:

47÷9=5 with a rem. of 2; 22−5=17;
17÷9=1 " " " 8; 81−1−15=65;
65÷9=7 " " " 2; 26−7− 3=16;
16÷9=1 " " " 7; 74−1−21=52;
52÷9=5 " " " 7; 70−5− 3=62;
62÷9=6 " " " 8; 80−6−15=59,
etc.

It is here seen that the same analysis hitherto employed will demonstrate the correctness of this method, and a casual inspection would lead the observer to conclude that the method by subtraction was by far the most preferable, since the divisor itself contained both the base and the whole difference; but repeated trial and experiment have demonstrated beyond question that for general use, the method by addition is the superior. In isolated cases the method by subtraction may be advantageously employed; but these instances lessen as the divisor increases. In general it may be adopted as a safe rule, that *the method by subtraction should only be employed when the digits following the first digit of the divisor are all below* 4.

The analyses of the two methods are identical, and the general rules similar, transposing the words "add" and "subtract." It would have been simpler to have analyzed the method by subtraction, but as the method by addition is the one for *general use*, the latter was chosen as the typical process of corrective division. An elaborate analysis of the method of subtraction is unnecessary, as its comprehension is assured to those who thoroughly understand the method by addition.

Corrective Multiplication.

As was first stated the method of corrective division was discovered in seeking an abridgement of the ordinary methods of multiplication and division. In reality the abridgement of multiplication was first discovered; but as the writer subsequently ascertained that this process had been previously employed, it is only included herein as an addenda to corrective division. Corrective multiplication is closely analogous, and in fact identical in analysis with corrective division. It may be briefly explained as follows:

Let it be required to multiply any number as 742137261 by any other number as 4325. Writing these numbers as in the ordinary process of multiplication, the product may be directly obtained by the following analysis:

742137261
4325

3209743653825

5×1, or the last digit of the multiplier $\times$ the last digit of the multiplicand, $=5$ = last digit of the product. $5\times6=$ second digit of the partial product to which must be added 2×1 or the second digit to the left of the multiplier $\times$ by the last digit of the multiplicand $=30+2=32$. Carrying 3, it is added to $(5\times2)+(2\times6)+(3\times1),=28$; $\therefore$ 8=third digit to the left, of the product; carrying 2, $2+(5\times7)+(2\times2)+(6\times3)+(1\times4)=63$; $\therefore$ 3=fourth digit to the left, of the product. Carrying 6, $6+(5\times3)+(2\times7)+(3\times2)+(4\times6)=65$; $\therefore$ 5 = fifth digit to the left, of the product, and the process is continued until the final partial product of 4 and 7, or the product of the first digits of the multiplier and multiplicand, + the number carried from the preceding step will give the first two digits of the entire product.

The analysis employed in corrective division is equally applicable in this method of multiplication. In the above example, 4 may be taken as the base and 325 as the whole difference, or 5 as the base and 432 as the whole difference, bearing in mind their relative connection.

It is also evident that the multiplication may be commenced by multiplying the first digit of the multiplicand by the first digit of the multiplier, but in this event a necessity would arise for constant correction.

The corrective method of multiplication may be profitably employed, though the saving of time and labor is by no means so marked as in the process of corrective division. It is given place herein to show the constant relation of the two general processes of multiplication and division, and to demonstrate how an analysis of one may invariably be applied to analyze the other.

While a still further investigation of the process of corrective division will result in the discovery of new methods for special instances, and the formulating of new rules for the student's guidance, it has been sufficiently analyzed and ex-

plained to enable anyone of ordinary intelligence to employ it in every case where long division has hitherto offered the only means of solution. In the foregoing examples its operation is amply explained and analyzed; and, in conclusion, the writer claims for the process which he has entitled "Corrective Division:"

1st. Economy of labor. The use of figures is greatly abridged, and the computations are of the simplest nature, being readily performed mentally.

2d. Economy of time. After a little practice—no more than is required in long division—any person of ordinary intelligence may accomplish the desired results in from one-fourth to one-sixth the time requisite in long division. Repeated trials by both experts and non-experts have clearly demonstrated that the corrective method is the quickest for both classes, the economy of time being proportionately equal in nearly all cases. By corrective division the result may be often obtained in almost as little time as it would were the problem one classed under short division, and when the quotient is desired to be carried out to a number of decimal places, the economy of time is still more marked.

3d. Originality. To the best of the writer's knowledge and belief the process of corrective division is herein described, analyzed and explained for the first time.

ELECTRICITY AS A MOTIVE POWER.

By PROF. GEORGE FORBES.

From the "Journal of the Society of Arts."

THE subject of the transmission of power by electricity has been so often discussed in this room, that I have had some difficulty in selecting the point of view from which I should look at the subject to-night. I presume that most of my audience have seen the action of a current which was generated by one dynamo acting upon another dynamo so as to cause it to rotate. I presume that they have also seen such a motor driving a circular saw, or a fan, or a sewing-machine. They may also have heard of the grand possibilities among which the imagination delights to revel, on the realization of which we shall look back on the old life without electricity as being but one step removed from barbarism. I propose to myself something more humble, but, perhaps for all that, quite as useful. That is, to discuss the results which have been already attained, and to show what steps are immediately possible with the knowledge and experience now at our disposal. I will not weary you with descriptions of machines, nor refer to their relative merits. In quoting the performance of any special machine, it will not be so much on account of its merit, as because it may have come more especially to my notice in connection with the particular subject which I may be talking of.

Of course, from the moment that Oersted discovered the action of a current on a magnet, it was known that energy could be transmitted to great distances by a wire. The energy which is put in at one end of a telegraph cable is partially reproduced thousands of miles away, in the form of mechanical movement. But what we have to consider is, the condition of using a current to supply motive power to drive machinery. In what cases is such an application practicable, in what cases is it economical, in what cases convenient? I wish to give you, this evening, an answer to these questions, and to explain what has been done and is being done in this field; how far we can build on past experience, and how far we are justified in foreseeing steps in the future. When new discoveries are made, we are distrustful of them; but we are too often, also, apt to be over-sanguine; we dive, in our imaginations, into the future, and conceive that that immediately will be accomplished which experience ought to warn us can only be the result of many decades of labor.

Let me give you an example. We are

constantly hearing at present of a certain method by means of which water-power, it is said, will be extensively used. This is to use it in charging accumulators with electricity, and then to carry the accumulators on a tramway or railway, and use them to drive the wagons. At first sight this seems very feasible, but reflection makes us pause. The same water-power has been always available: it might have been used to compress air; compressed air tram-cars have been constructed, and, in my opinion, are a perfect success. This method of application of the water-power is much simpler than by the electrical accumulators. But it has not yet been tried, and hence we cannot hope that this method of using electricity is likely to be very soon introduced.

From the time that it was known that a current of electricity, in going through a wire round a rod of iron, converts the iron temporarily into a magnet, inventors have been at work trying to utilize this knowledge in the construction of motors. The chief difficulty lay in the very short distance through which the attraction of a magnet remains powerful. There was also, in the motors of old date, a difficulty of getting over the dead points. The length of pull of a magnet may be considerably increased by various devices. A somewhat similar action may be effected by utilizing the suction of the current through a coil of wire upon a rod of iron. A coil of wire with a current through it acts as a magnet. One end of it attracts the N end of a magnet. But the coil also magnetizes a rod of soft iron placed in this position, and, having magnetized it, it attracts it.

This principle has been utilized directly to obtain power from electricity. The most interesting application of it which I have seen, was at the Electrical Exhibition at Munich last year. That exhibition was especially of interest owing to the various applications of electricity as a motive power, and I shall have occasion to refer to it more than once. The machine to which I now refer is designed by M. Deprez, and is called an electrical hammer. It is intended to perform the same functions as a steam-hammer. It consists of a vertical tube containing a short rod of iron, and surrounded with a coil of wire. The wire at each turn is led away to a contact piece on a plate with a movable switch. By this means, the current passes only through fifteen turns of the wire. But there are one hundred tons of wire altogether, and by moving the switch you can pass the current through any fifteen you please. You suck the iron into one section, and then move the switch, and so raise the center of attraction. In this way you suck the iron up to the top of the tube, and then you can equally well drive it down, with the force of suction added to the weight of the iron. The weight of the iron was 80 lbs., the height of the tube one meter. I can illustrate the principle with this model. He used a current of about 36 amperes; 31 amperes was found sufficient, when passing fifteen times round, to support the weight of the iron. The length of the iron rod was about six times its diameter. This is equivalent to saying that, in such a coil, a current of one ampere must circulate (15×30^2), or 13,500 times, to support the iron rod in the centre. I tried independently with this small coil, which has three layers of forty-three turns each (129 in all), and I found that five amperes were required to support the iron in the center. From this result one ampere would have to make 16,200 turns in place of 13,500, as Deprez found. I give these data because they help to fix our ideas, and numerical measurements in magnetism are far more scarce than in electricity. Dr. Werner Siemens has used the same principle in the construction of a motor to be driven by electricity, though I need hardly say his invention was not derived from the one just mentioned. It will readily occur to you that the electrical hammer of M. Deprez might, by a simple modification, be converted into a reciprocating motor, like the piston of a steam engine, and so it might be used to drive a fly-wheel and machinery. But in this mode of applying it, the whole of the momentum of the iron mass would be lost at the end of each stroke. The same is true of the piston of a steam engine. But in the rotary engine it is not the case. Now although the rotary engine is not so economical as other steam engines, its principal serves to lead us from the electrical hammer of M. Deprez, to the beautifully ingenious electric motor of Dr. Werner Siemens. If we suppose the tube with the coil of wire round

it to be bent round into a circle, the iron cylinder being also bent into an arc of a circle, then, by changing the points of contact for the current to pass through the coils, we could suck the iron core round and round inside the circular tube. This is the principle employed by Dr. Werner Siemens in the elegant motor which he has lately produced. The inner iron core is made in the form of a complete ring, but only a certain arc is of iron, the rest is of brass. It rests on wheels, spaces being left in winding the coils for these supports to reach the central ring. In the actual machine, the points of contact for the current to enter and leave the coils are changed automatically by the rotating ring. In such a form of motor, the magnetism of the iron is never changed or destroyed, and this diminishes largely the waste of energy, by heating during magnetizing and demagnetizing, which is so serious an objection to most motors—I think I might safely say to all other motors.

If this form of motor were used to drive machinery, it is easy to see that, instead of making contact automatically, it would be quite easy to do so by hand, simply turning a handle round; and by doing so more or less rapidly, the speed of the machinery might be regulated by a boy with most perfect facility. It could be quickened, slowed, stopped, or reversed, by the simple process of turning the handle. I consider that there will be a great demand in the future for such a form of motor. I will give one example. I am sure many of my audience will sympathize with me when I speak of one of the horrors of a sea voyage, I refer to the terrors of the steam-winch. What traveller has not spent sleepless nights under the constant influence of the whirring and jarring of that dreadful engine when stowing or discharging cargo. For many years I have felt convinced that this was a field for the application of electricity as a motive power. But for a long time no scheme suggested itself to me which was perfectly satisfactory. In fact, it was not until I saw Dr. Siemens' motor that I felt satisfied that the thing could be accomplished. I have no hesitation in saying that such a motor, with the contacts made by hand in the manner I have described, will be a very perfect machine, and very much more easily managed than the present steam-winch, with its troublesome combination of valves and brakes of multiplying and reversing gear. When such machines are introduced, life on board ship will be a different thing.

Now, I am not going to spend time to night in describing all the different motors which have been invented. Most motors now in use are simply generators of electricity used in a different way. Many years ago, after a prolonged absence from England in the Pacific, I was writing to the late Professor Clerk Maxwell, and asked him what had been going on in science during my absence. His answer was, "the greatest discovery of late years is that a Gramme machine can be reversed." He saw that this must be a more perfect type of electro-motor than any previous one, and that now there was a real future for employing electricity as a motive power. The Gramme machine was a cheap generator of electricity, and was an efficient motor. The problem was solved, and only required development. Of course, Professor Clerk Maxwell's remarks would have applied to any other of the modern generators. I am not going to trouble you with a description of these machines; their construction is known to most of you. It suffices to say that, when one machine is turned so as to give a current, it can drive another machine at a distance.

Now let me say a few words about the convenience of electrical motors. We are certainly a long way from having electricity laid on to our houses, like water, which we may use for any purpose we please. But when it is laid on, it is certain that many people will use it, because they now use water-engines, although water is very expensive, and because electric motors would be far more convenient in many ways. But in a large factory, the only methods available for distributing power is either by shafting or by steam pipes. Both of these are objectionable in many ways, and the convenience of having only to lead wires to the different places where power is required, will in time become manifest to many of our manufacturers. This remark applies more especially to donkey-engines and steam steering apparatus on board ships; and I confidently look forward to electricity being largely used for

the light work on steamers, and this before any long time. Of course, special attention will be given to see that the compasses are not affected by the powerful magnets and currents used for these purposes.

It is found that to get any good effect an electric motor must run at a high speed. In this there is a great difference between the theory of electro-motors and of steam engines. If a steam locomotive were restrained from running down an incline by the pressure of steam on its piston, the steam would be doing no work, and no power would be used up, except by a slight condensation. But if an electric locomotive were to perform the same office, it would be using up the current all the time, and yet not performing external work. When with the electrical hammer the iron cylinder is supported in the tube by the electric curaent, power is being used up, and yet no external work is done. It is the same as with muscular exertion. This introduces us to a new factor in mechanics, the cost of the statical effort. When I support a weight, I am using up muscular energy, and yet I am doing no external work.

But there is one peculiarity in electric machines of great importance. The motor is also a generator. Thence it follows that, when it is in motion, it creates an electromotive force. This tends to produce a current of electricity in the circuit, going in the opposite direction to the one which is driving the motor. The intensity of this opposing force is directly proportional to the velocity of rotation. If the generator and motor were identical, and the work done by the motor was so far reduced as to allow it to rotate nearly as fast as the generator, then the opposing electromotive force would almost be equal to that of the generator, and there would be hardly any current in the circuit. But if a brake be put upon the motor, more work is done, and the current in the circuit increases. The maximum amount of work is got from a motor in this way when the counter-electromotive force is equal to half that of the generator. The greatest efficiency (or return) is obtained when the counter-electromotive force is nearly equal to that of the generator. Some persons have argued from this that we ought to run motors with very light loads for the sake of economy. This is not the case. If the motors were run so as to do only half of the maximum, we should require twice as many motors, and usually the prime cost of the motors is too great to cause this to be economy. The efficiency of a generator for electric lighting or other purposes is highest when there is a very high resistance in the circuit, but electricians, as a matter of fact, nearly always work their dynamos up to their maximum capacity, although this is by no means the most economical, so far as the efficiency is concerned.

I have said that, to produce a good mechanical effect with a dynamo used as a motor, it is necessary to drive it at a high speed. This speed is generally too great for ordinary mechanical applications. For example, we do not require the wheels of a tram car to turn at anything like a speed of 800 turns a minute. In such applications, then, it becomes necessary to reduce the speed by suitable gearing. Thus, in the electrical tramcars which have been constructed hitherto, belting has been used, or some equivalent. This is an awkward expedient, and one which, in practice, is found to give much trouble. I am inclined to think that, in making an electrical railway, it would be preferable to make the wheels of small diameter and attached directly to the axle of the motor. This seems to be by far the best plan in the present state of affairs. It may be that a motor will be found which will work economically at a lower speed; but with those which are now at our disposal, it would be best to work with very small wheels.

We now come to consider, directly, the transmission of energy to a distance. There are two separate kinds of application. One is to fixed motors, the other to locomotives. Let us begin with the stationary motors. There is no doubt that if electricity be capable of transferring the water-power of rivers to the centers of mechanical industry, if it be capable of introducing economy by burning coal at the pit mouth, or the bottom of the pit-shaft, then we have a vast engineering industry ready to open out.

It is not until we have sat down and worked out from experimental data the cost of installation and maintenance, that

we can pronounce an opinion on the merits of such schemes. When we want to send a large quantity of energy to a distance, through a thin wire, we must use the highest electromotive force which we consider safe, and the most powerful current which the conductor will bear without either fusing or injuring its supports by heating, and then we have no means of increasing the amount of energy which we can send, except by increasing the size of our conductors. Unfortunately, when we double the weight of our conductors, we do not double the current which we can pass through them. Hence there must necessarily be a limit to the quantity of energy which can be sent along a single line, which limit is reached when the cost of the conductor becomes too great. When a limit is thus obtained, the only way of increasing the transmission of energy economically is by increasing the number of systems of conductors, or else by introducing means of cooling the conductors. These are the considerations which have guided M. Deprez in his experiments. But Sir William Thomson has shown that the loss of horse-power in heating the conductor more than compensates for the expense of thicker conductors. For economy, we ought to have the value of horse-power thus wasted equal to the value of interest and depreciation of conductors.

In such an installation we require to consider not only the quantity of power which can be transmitted, but also the efficiency of the arrangement. Now the efficiency varies with the quantity of power which is transmitted. If we transmit very little energy, we get a high efficiency, nearly $= 1$; if we transmit the maximum amount of energy possible, the efficiency is $\frac{1}{2}$. In fact, the work done by the motor is proportional to the efficiency minus the square of the efficiency, where $w = 4M(e - e^2)$.

M = the maximum work possible.
e = the efficiency.
w = the actual work returned.

and consequently, when the efficiency is nearly equal to unity, the work done is zero. It is found in practice to be preferable to get nearly the maximum work out of a machine, rather than to increase the size and cost of the machines, in order to get a certain amount of work from the apparatus.

Of course, it is only after long experience that we shall know what is the maximum power which we can send along a wire, and what limits must be put to the electromotive force which we use. M. Marcel Deprez has spent much time and labor on making practical tests of this nature. One of these formed a chief attraction at the Munich Exhibition last year, where he used an iron telegraph wire 4 mm. diameter (or a sixth of an inch), and transmitted $\frac{1}{2}$-horse-power to a distance of $35\frac{1}{2}$ miles. He has since been carrying on experiments of the same kind at Paris. The results have lately been published, and I shall return to them.

One of the chief things which I proposed to myself, when I was asked to read a paper here, was to give some account of the experiments which I saw at Munich. I was very much struck indeed with the transport of energy by Mr. Schuckert, of Nürnberg, not that there was any new principle involved, but I had never before seen so much as five or six horse-power conveyed by a thin wire so far as three and a-half miles. The power was derived from a turbine, at Hirschau, beyond the English gardens. These turbines are employed on a locomotive factory, no steam being used in any of the shops. They have thus 260-horse power at their disposal. The machine at Hirschau was of a larger size than the motor in the exhibition. This is a point on which Mr. Schuckert has made careful experiments. This machine which was driven by the turbine, worked a number of agricultural machines during the day, and served to light thirteen Pilsen lamps at night, both being in the exhibition building. The turbine being a very large one, and revolving slowly, transmitted its power to the dynamo by belts. A turbine is a very convenient motor for a dynamo, for it happens that, when we are using a dynamo requiring the whole power of the turbine, the proper speed for each is nearly the same, so that the dynamo may be attached directly to the shaft of the turbine, which is, of course, a very great convenience, and a saving of power. In my experiments, in conjunction with Dr. Young, on the velocity of light, at Wemyss Bay, I made use of such an arrangement for producing the electric light. A small

Siemens machine was attached directly to a Thomson's vortex turbine with horizontal shaft. The water was led from a distance of nearly a mile. The convenience of being able to start the electric light by merely turning a handle was very great indeed, and I can recommend this arrangement to anyone who has water-power at his disposal.

But to return to the Hirschau installation. A pair of No. 7 copper wires conveyed the current, one-eighth of the E.M.F. being wasted in heating the wire. This installation, taken as a whole, was much to be admired. I do not say that there was anything new or much information to be gained from it, but it showed to everyone visiting the exhibition a good working practical installation. Mr. Schuckert has been good enough to give me the results he has obtained from two experiments with different sets of machines. All the four machines with which these experiments were made are constructed for generating a current of eight ampères. In one experiment the generator was constructed for fifty-five incandescent lamps, with a motor constructed for thirty-three lamps (the lamps being in compound parallel). In the other he used as generator a machine for about 150 lamps, with a motor made for eighty-eight lamps. The particulars of the machines are given in the following table, from which we see that in each case 36 per ecnt. of the energy put into the generator was reproduced as work done by the motor:—

	First Experiment.	Second Experiment.
Type of generator.....	TL 3	TL 5
Type of motor.....	TL 2	TL 4
Difference of potential—		
(1) At terminals of generator	300	700 volts.
(2) At terminals of motor	260	600
Resistance of leads.. ...	6 ohms.	12 ohms.
HP. spent on generator..	4.4	12.8
HP. given by motor.....	1.6	4.7
Efficiency (per cent.)....	36	36

I give you these figures, not because they show any wonderful performance, but because it is desirable to have as many facts as possible when any new idea is being developed.

You see that, in these experiments, the aim has not been so much to get a high efficiency as to get a high amount of power developed. This is what ought always to be done, while the machines form the chief item of cost.

Let us now speak of the experiments of M. Marcel Deprez. In 1881, M. Deprez published the theory of transmission of energy to a distance, by means of a thin wire. It is a theoretical fact that, if you get a certain return by using a given generator and a given receiver, and with a certain length of wire between the two, you can double the distance with the same results as to power, using the same thickness of wire if you construct a new generator and a new receiver wound with smaller wire, so as to have double the resistance of wire wound, the other dimensions of the machine being unchanged. This had been known before, and the theory had been fully worked out by Frölich and others. But Deprez had the credit of working out numerical examples, and of afterwards putting the theory to the test of experiment. In his first paper, he started with some experiments made at Chatham, in which two Gramme machines were used, one as generator, the other as receiver, when, through a line-resistance equal to that of either machine, a return of 50 per cent. was obtained. He then showed that theoretically a similar pair of machines could be constructed so as to transmit 50 per cent. of the energy to a distance of 100 kilometers, an ordinary telegraph wire being used, 4 mm. thick, giving a resistance of 9 ohms. to the kilometer. He also stated that 12-horse power being used by the generator, 6-horse power would be delivered by the receiver. I do not consider that the theory in this paper was quite clear, but the main point was proved.

After the publication of this paper, the Munich Exhibition was a thing of the immediate future, and M. Deprez was invited to make his experiment with a distance of sixty kilometers. The resistance of a telegraph wire of this length, using the earth for the return, was 475 ohms. Consequently, two machines were made of the normal type, like an "A" Gramme, each of 453 ohms resistance. These machines developed very high electro-motive forces (about 2,000 volts), and when they were installed, it was naturally discovered to be impossible to use the earth as a return. Consequently, another wire was used, increasing the resistance to 950 ohms

This militated against the complete success of the experiment, and I think it is unfortunate that, in their desire to assert the success of the experiments, the friends of M. Deprez withheld this fact from the public, and so led many people to believe that the theory was inexact. As a matter of fact, owing to the great resistance of the line, 60 per cent. of the total electric effect was used up in heating the circuit, instead of 40 per cent., as had been calculated. In order, then, to get a satisfactory return, it was necessary to allow the receiver to do very little work. Thus, in the trial made by the committee of experiments before the machine broke down, only 0.433 of a horse-power was utilized at Munich, and there was a return of 39 per cent. of the power put into the machine at the distant station of Miesbach.

These experiments were of a very high degree of interest, and M. Deprez deserves the thanks of scientific men for the zeal he showed in carrying them out. Let us see, now, whether an economy would thus be introduced by burning coal at the pit mouth, and transmitting power, as in the Miesbach experiment. Allowing 3 lbs. of coal per horse-power per hour, if we work twenty-four hours a day for 300 days in the year, we have almost exactly 10 tons of coal per horse-power per annum. Suppose the coal costs 5s. a ton at the pit mouth, and 1½d. per ton for transport over the kilometer, it follows that the cost of furnishing 0.433 of a horse-power at Munich by transport of coal would be the cost of 4.33 tons of coal, and their transport over 60 kilometers, amounting to £1 11s. 6d. The cost of coal to work the machine at Miesbach, with a duty or efficiency of 39 per cent., would be £2 19s. Let us assume the machines to cost £100 together, then, neglecting the cost of the telegraph wire, and allowing 15 per cent., for interest and depreciation of the machines, neglecting also the extra cost of the larger steam-engine required at Miesbach, we have an annual expense of £17 19s. for the power delivered electrically at Munich, against £1 11s. 6d. for the same power delivered by the transport of coal. Even if the machines cost nothing there would be a loss; if the power cost nothing there would also be a loss. It is folly to use machines costing so much only to produce ½-horse-power.

I have no doubt that M. Deprez could have got more than a ½-horse-power at Munich, but had he attempted this it would have been at the cost of efficiency. It cannot be too constantly borne in mind that, in electro-mechanics, you can increase your efficiency almost up to unity, but at a sacrifice to the total amount of work transmitted. But it must also be constantly remembered that you increase your total work up to a certain maximum, but at the cost of efficiency. The maximum efficiency is obtained when the load is so great that the motor is only just able to turn round. The efficiency is greatest when the load is so light that the motor turns nearly as quickly as the generator. I repeat that an erroneous idea has been promulgated that we ought always to run motors at a speed approaching to this condition of maximum efficiency. But if this be done, the return of work is insufficient to pay for interest and depreciation on the plant required.

The very important experiment of M. Deprez at Munich, is able to give us much information for future use. Among other facts, we learn that, with machines of the type described by him, 2,000 volts is too high an electromotive force to employ. Tue reason is that it is impossible to prevent the contact of the brushes from being sometimes accidentally broken. In such a case it was found that the electromotive force developed by the extra current is sufficient to ruin the insulation.

Another matter deserving of attention is the relative size of the generator and motor. M. Deprez employed two machines of equal size. This slightly simplifies the theory, but it is certainly not the most advantageous arrangement, and we are much in want of accurate measurements on this head.

From what has been said, it is clear that the Deprez experiment at Munich was, commercially, a failure. You might, at first sight, think that if a thoroughly scientific experiment like this was a commercial failure, there are no cases in which it would be a commercial success. This is by no means the case. Since the time of the Munich Exhibition, M. De-

prez has been occupied with experiments of a far more practical nature, with a line of 160 ohms resistance. In the latest experiments, where the effects of friction were deducted, a return of 47½ per cent. was obtained, and 4.4 horse power of work was actually given off by the motor. Judging from the Hirschau experiment, I should say that if a fall of water be used as the motive power, we can install a turbine and dynamos which shall transmit 6 horse power through a resistance of 12 ohms, at a cost of £200, neglecting the unknown cost of the conductor. If this power were used in a place where coal costs 20s. per ton, the cost of fuel for 6-horse-power would be about £60 per annum. The interest and depreciation on the boiler and steam-engine would be about £30 per annum, making in all £90 per annum, exclusive of wages. Electrically transmitted, the interest on plant, at 15 per cent. would be £30 per annum, exclusive of wages. This difference of £60 per annum, after deducting from it the interest and depreciation of the conductor, is so enormous, that it is easy to see what a large saving would be effected in any installation where there is a large consumption of power. There are many factories where it is essential to use a high priced coal, but if the power could be conveyed electrically from a distance of a few miles, an immense saving would be effected by employing a cheaper kind of coal. When water-power is used, it often happens that we can convey the cables along the bed of the river. This preserves the insulation, and keeps the conductor cool, so preventing the usual increase of resistance by heating.

In some towns, and notably in Sheffield, the whole of the water supplying the town comes from reservoirs at a very great height. The very large quantity of energy of this water is at present absolutely wasted; and although coals are not dear in Sheffield, and manufacturers are permitted to burn the most smoky kinds of coal in the center of the town, yet the employment of this waste water-power would add very considerably to the commercial prosperity of the town, and the water would be injured in quality no more by passing through turbines than by passing through the present pipes. If this power were distributed electrically, the benefit to the community would be great indeed.

At the site of the Severn Tunnel there is a width of river of two and a-half miles, where the average rise of tide is fifty feet. If the average rate of flow across this section were one mile per hour, we could utilize 100,000 horse-power, and the market value of that power is something like £1,000,000 per annum, which is now allowed to be wasted. It is worthy of the most serious consideration whether it would not be worth while to erect the enormous engineering works which would be required to utilize this wasted energy, or rather a portion of it. Assuming the interest and depreciation on the turbines and dynamos to be at the rate of £2 per annum per horse-power (and it would be far less than this for a large installation), it is easy to see that a time will come when this source of wealth will not be allowed to be wasted.

I wish to draw special attention to this point, because I have frequently heard people say, with regard to the utilization of the tides, "Oh! that is out of the question: Sir William Thomson has proved that the area required to produce 100 horse-power is so great that it would be more worth while to reclaim the land and cultivate it." Now Sir William never said such a thing. It is true that at the York meeting of the British Association, he showed what area would require to be enclosed in order to produce 100 horse-power on the supposition that there was a rise of tide of only 6 feet 7 inches (2 meters). And he made use of the following expression:—"Thus we are led up to the interesting economical question, whether is forty acres .... or 100 horse-power more valuable." But in the case I am considering, the height of the tide is six times that contemplated by Sir William, and the question is rather "whether is forty acres or 600 horse-power more valuable." The estimate I made above, as to the value of a horse-power, agrees with Sir William's; and the value of 600 horse-power is £6,000 per annum. I confidently state that the reclaimed land on the Severn would not be worth £150 per acre per annum. I have taken the site of the Severn Tunnel as an example, but there are many other places available,

and I have little doubt that the industrial population at Bristol will soon realize the wealth of power at their doors.

Some people, in this country, are timid about the introduction of large turbines. This, I need hardly say, is a gross prejudice. At Connecticut, Massachusetts, 30,000 horse-power is derived from the river by turbines. At Narva, near St. Petersburg, one firm uses 12,000 horse-power, and the size of the turbines used, which were made at Augsberg, was limited only by the size of the tunnels on the railways by which they had to be conveyed. At Munich I saw 260 horse-power rendered available by means of turbines with a fall of less than two meters. In the remote valleys of Italy there are several installations of incandescent electric lighting by means of turbines driving the machines of Messrs. Crompton. In Scotland, too, we have powerful turbines. Mr. Pirie, the paper-maker, has introduced, near Aberdeen, turbines working up to nearly 1,000 horse-power. At Greenock large overshot wheels are employed. The Shaws Water Worsted Company are now replacing one of these, of 200 horse-power, by a turbine. In Lancashire water-power is also extensively used. At Valserine, on the Rhone, a canal has been cut through the rock, a third of a mile in length, thus giving a fall of about 12 meters and many hundred horse-power, using Jonval turbines. It is proposed to use the falls at Bellegarde, near Geneva, in the same way. The Manville Cotton Company, at Albion, R. I., use turbines, with a fall of 5.4 meters, which takes the place of a steam-engine of 800 horse-power, and economy is materially gained, although during the five months of the dry season they are obliged to use steam. At the Pittsburg Mill, Minneapolis, there is a Victor turbine, working up to 1,400 horse-power, and another is being added of the same dimensions. These are only a few examples to show how thoroughly practical, economical and convenient the use of turbines has become. We can with them calculate on utilizing 75 per cent. of the theoretical power of the head of water. They can be controlled with the utmost ease. Their cost is very small compared with water-wheels, and they can be geared directly on to the shafts of dynamos. As an example of their adaptability to the purposes of the electrician, I may mention the performance of a Fourneyron turbine established at St. Blaise in the Black Forest, which utilizes a fall of 108 meters, turns at a speed of 2,300 revolutions a minute, and gives off 60 horse-power.

The engineering works which would be required on the Severn would be very simple, though on a large scale, and the action could be made quite continuous, independently of the state of the tide, and without any necessity of using accumulators.

Having now spoken of the applications of electricity to stationary engines, let us see what has been done, and what can be done, with respect to locomotives. There are two ways in which this application has been made. First, by storing the electricity in accumulators, and putting the accumulators in a tram-car, a boat, or a tricycle, and second, by deriving the current from a wire or other conductor placed along the line. The experiments made upon the first plan are very interesting, but their practical value is limited, owing to the weight and expense of the accumulators. I think that it would be far more economical to use compressed air engines. Ever since 1865, when I examined the compressed air tram-car of Scott Moncrieff, which ran so successfully in Glasgow, I have felt sure that this must become eventually the motive power for tram-cars, and I have not yet seen anything to lead me to alter my opinion.

What has been done in the second way of proceeding? We will see that considerable progress has been made. At the Berlin Exhibition in 1879, Messrs. Siemens and Halske drove a train of a few carriages round a circular tramway, the gauge being three-quarters of a meter. In April, 1881, the same firm constructed the Lichterfelde and Berlin electrical railway, which is two miles long. One of the carriages of this railway was shown at the Paris Exhibition of 1881. The current is passed through the rails, which are insulated. It is then taken up by the wheels, and so passed through the motor. The motor turns at a very rapid rate, but it is geared to the wheels of the carriage by belting of wire rope, which reduces the speed of revolution. The carriage has the appearance

of an ordinary tramway-car, and the machinery is invisible. It was expected that snow might prove troublesome in such a system where the current is passed through the rails, but the winter of 1881–82 was so mild as to give no trouble. I have not heard whether any difficulty presented itself during the past winter. At the Paris Electrical Exhibition a somewhat similar car was used, but Messrs. Siemens were not allowed to pass the current through the rails, on account of the danger to horses, &c.; hence they were obliged to carry conductors, consisting of split tubes, on telegraph poles, and a small carriage fitting into the tubes made electrical contact with the motor on the car. Another electric railway has now been completed by the same firm, between Charlottenburg and Spandauer Bock. In this case, also, a contact carriage is used, as at Paris. This line is three and a half kilometers long; two and a half kilometers are level, but this line is specially interesting from the fact that a whole kilometer is on the considerable incline 1 in 28. Messrs. Siemens have also applied the same principle to haulage in mines. A small electrical locomotive is used in a mine near Dresden to drive eight corves loaded with coal. Finally we have the electrical tram-car running on the Portrush line, in the North of Ireland, but this has been so well described in this room, and so recently, that I need only mention it.

It has always seemed to me that the principal advantage which would be gained by using electricity as a motive power for railway trains, lies in the great facility with which motive power could be applied to every wheel of the whole train. Captain Douglas Galton, in the concluding paragraph of his report on experiments conducted by himself and Mr. Westinghouse, on continuous brakes, says:—"The advantage which thus evidently ensues from utilizing the adhesion of every wheel of a train, suggests the further consideration as to whether it would not be a more scientific arrangement, as well as being more economical in regard to the permanent way of railways, to utilize the adhesion of every wheel of a train for causing the train to move forwards, instead of depending for the moving force upon the adhesion of one heavy vehicle alone, viz., the locomotive."

Certainly, there can be no doubt that it would save the permanent way from tear and wear, and the passengers from the shocks which are so unpleasant. But it is only by means of electricity that the proposition is practical. Moreover, the movable parts of a locomotive are not perfectly balanced, and this limits the speed at which it can go. With an electrical motor the balance is perfect. By utilizing a motor on every wheel, you could go safely round curves at far greater speed. I feel sure that in this way it would be safe for trains to go at double the speed they do at present, and I hope to live to travel from London to Edinburgh in three and a half hours, smoothly, and without jolting. Furthermore, a train, instead of taking several minutes to get up full speed, could do so in a few seconds. This follows from the same reasoning as is applied to continuous brakes. Lastly, if there were power enough, a train could go up a steep hill instead of being confined, as practically they are, to an inclination of 1 in 80. In a district with heavy goods traffic, this might be a great advantage.

These considerations appear to me to be so important, that I must pause a while to make them clear.

First.—As to getting up speed quickly. With an ordinary train, weighing ten times as much as the engine, the theoretical minimum of time required to get up a speed of thirty miles an hour is one minute, however powerful the engine may be. The reason is that the wheels would slip if the motion were accelerated more rapidly. But if power were applied to every wheel, it would have a speed of 30 miles an hour in six seconds, and with no more jerk than is now felt when a continuous brake is applied.

Second.—As to the going up hill. The same advantage here is gained as would be gained by continuous brakes. An ordinary train, when the lines are in an average condition, slips down an incline of 1 in 50, when the locomotive alone is braked. You all remember the sad accident the other day in the Rocky Mountains from this cause. But when all the wheels are braked, the train rests on an incline of 1 in 5. So also, if we had the power, we could ascend an incline of 1 in 5 by electrical motors.

Third.—As to advantage to permanent

way and speed. The engine does not give a steady pull at all times to all parts of the train. In fact, in going round a curve, part of the engine's power is used up in trying to pull part of the train off the lines. With power applied to all the wheels, this would not be so. Moreover, the enormous weight of the locomotive, imperfectly balanced, which is the chief source of mischief, would be done away with.

Fourth.—As to cost. The cost of making an ordinary railway line with tunnels, embankments, bridges, viaducts, &c., in England, may be taken at £15,000 a-mile; but if it goes up inclines, instead of having expensive engineering works, the cost will be the same as making it on a level, *i. e.*, £5,000; and hence, on a goods line, the cost of construction might be reduced to one-third. This original outlay is a far more serious matter than the working expenses of a railway (of which the cost of the traffic is 26 per cent.); so that, even if by using electricity you increased your working expenses, you might still be making a great saving on the whole. On fast passenger trains, the power to drive your trains at twice the speed would counterbalance any small loss of economy. Again, there would be a great saving in the cost of maintenance of permanent way.

There are many cases where such an application of motive-power would be peculiarly satisfactory. The most important of these is the Underground Railway in our city. It is now universally admitted that steam locomotives vitiate the air to an intolerable extent. If motive power were used electrically, this evil would be avoided. Moreover, a line like this, which is almost totally underground, and is comparatively unaffected by snow or rain, possesses exceptional advantages for electrical insulation. But the system which I have advocated would introduce special advantages to this line. What are the conditions of working? The distance between stations is so short that a great deal of time is wasted in getting up speed and stopping the trains. The latter evil, it is true, has been reduced to a minimum by the adoption of continuous brakes; but you will understand how much speed is limited by the former consideration, when I tell you that an ordinary train cannot get up a speed of thirty miles an hour until it has passed over a quarter of a-mile, just about the distance between some of the stations. If the system I propose were adopted, of applying motive power to every wheel, the same speed would be attained in forty-four yards.

The Channel Tunnel, when completed, will be specially favorable for the insulation of conductors. But the system which I recommend could not be adopted, as the great advantage of the tunnel is that the same wagons will go right through without any transference of goods. In this case, if electricity be used as the motive power, it would be necessary to use it to drive a heavy locomotive.

I think I have now said enough to prove that a great deal has already been done, and that we may reasonably expect a great deal more to be done in the use of electricity as a motive power. Water-power has been usefully employed in various installations to obtain electricity, and the current has been successfully conveyed several miles, to be reconverted into motive power with great economy, and electric railways have been successfully and economically worked at several places. I have not thought it right in this paper to indulge in Utopian dreams of what may be done in the future, however pleasant such an exercise of the imagination might have been. I have thought it to be more useful to look at the matter from a practical point of view, and to estimate our present position with respect to this force, and to indicate the steps which our past experience would warrant us in taking in the immediate future. I should like to see a larger number of our wealthy compatriots experimentalizing in this field, so as to keep our country in the front rank of the applications of electricity, as she always has been in those of steam. But we must go on step by step—learning from every fresh experiment, and utilizing experience for further advances. We must not hope to reach perfection at one bound. The development of this most potent of agencies must, in the nature of things, be gradual. We must have patience, but at the same time we must work and labor to our utmost, and

take a step onward wherever that step is sure, and in due course we shall be ourselves astonished at the progress which has been made.

"We have not wings, we cannot soar,
But we have feet to scale and climb,
By slow degrees, by more and more,
The cloudy summits of our time.

The heights by great men reached and kept
Were not attained by sudden flight,
But they, while their companions slept,
Were toiling upward in the night."

REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF MECHANICAL ENGINEERS.—The annual meeting began in Cleveland, O., June 12, with a large attendance. The members were welcomed by Mayor Farley and the president, Mr. E. D. Leavitt, Jr., made a brief address, when the Society proceeded at once to business.

Mr. J. F. Holloway, of Cleveland, then read an elaborate paper on the Marine Engines of the Lakes, embodying a history of the development of marine engineering as applied to lake steamers, and a description of a device for getting them off the dead centers.

Mr. Howell Green, of Jeanesville, Pa., then read a paper on the development of the Winding and Pumping Machinery of the Anthracite Coal Regions, which was of much interest.

This was followed by a paper on Economy in Lubrication of Machinery, by Mr. George N. Comly, of Wilmington, Del. This paper called out some discussion.

After adjournament the members were entertained by the Cleveland Civil Engineers' Club.

SECOND SESSION.

On the second day reports were presented by the Council, the Treasurer and the Tellers, and a number of new members were elected.

Mr. W. F. Durfie, of Bridgeport, Conn., then read a paper on Balancing Vertical Engines, which was followed by a lively discussion.

Mr. W. E. Ward, of Portchester, N.Y., read a long paper on Beton in Connection with Iron as a Building Material.

A paper by H. R. Towne, of Stamford, Conn., on Cranes, was then read. This was followed by an address on the same subject by Mr. Thomas R. Morgan, of Alliance, O., who promised a future paper.

THIRD SESSION.

On the third day there was a long discussion on Mr. Towne's paper on Cranes.

A paper on the Bower-Barff Process for Protecting Metals was read by George W. Maynard, of New York, and called out a lively discussion, in which many members took part.

Papers were then read by Wm. J. Baldwin, of New York, on Standards in Pipe Fittings, and by Wm. Kent, of Pittsburgh, on Relative Values of Bituminous Coals.

Mr. C. C. Collins, of the Stearns Manufacturing Company, Erie, Pa., followed with a paper on Balanced Valves; and Prof. J. B. Webb, of Cornell University, on Reuleaux's Kinematic Models.

Before adjournament, Mr. J. F. Holloway, announced that he had received a dispatch from Prof. R. H. Thurston, of the Stevens Institute of Technology, which at its commencement this week conferred the honorary degree of Doctor of Engineering on E. D. Leavitt, Jr., President of the American Society of Mechanical Engineers. This announcement was received with much applause. After a brief reply by the doctor, the meeting adjourned.

In the evening, the members attended a reception tendered them by citizens of Cleveland.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The fifteenth annual convention began at St. Paul, Minn., on Tuesday, June 19, with a large attendance. The convention organized by the choice of George S. Greene as temporary chairman.

Addresses of welcome were made by Governor Hubbard, of Minnesota, and Mayor O'Brien, of St. Paul. These were responded to by Chairman Greene.

A permanent organization was then made, with Mr. D. C. Sheppard, of St. Paul, as Chairman, and the convention proceeded to business.

A paper on Building the Dyke at the Falls of St. Anthony, was read by Col. Farquahar, United States Engineers. This was followed by a paper on the Cost of Steam Power, by Charles E. Emery.

At Wednesday's session the two papers read on the previous day were discussed at considerable length.

Papers by Prof. Egleston, of Columbia College, on Accidents to Steam Pipes from the use of Slag Wool; by John Lawler, on a Pontoon Bridge over the Mississippi; and by G. Lindenthal, on the Rebuilding of the Monongahela Bridge at Pittsburgh, were read and discussed.

On Friday a session was held at Minneapolis.

On Saturday an excursion was made to Stillwater.

The attendance was larger than at any previous convention, 175 members being present, accompanied by 90 ladies.

ENGINEERING NOTES.

AN ANCIENT BRIDGE.—An interesting relic of antiquity was lately received at Berlin from Mayence, consisting of the remains of piles belonging to the bridge which once led from Castel to Mayence, and which is proved to have been in use fifty-three years before the Christian era. The pieces of wood are trunks of various trees, including oak, elm, and white beech and red beech. They are said to be internally sound, and to have pieces of iron at one end. It is intended to devote part of the wood to the manufacture of a piano-case.

The German papers state that the acquisition of these remains was difficult, as English collectors were offering high prices for such objects. Prince Alexander of Hesse has had

some ornamental furniture made from oak which was discovered at the spot referred to, which he has presented to his son, Prince Alexander of Bulgaria.

THE FORTH BRIDGE.—On Juue 7, Thomas Tancred, one of the contractors for the Forth Bridge, laid, with masonic honors, the first granite block of stone in connection with the structure. He was accompanied by Mr. Symons, one of the Government inspectors; Mr. Gray, manager of the works at North Queensferry, and several other gentlemen. The stone of pure Aberdeen granite, weighing about 16 cwt., was placed at the south-west corner of pier No. 12 from the south end of the bridge, which is situated a little to the east of the church at North Queensferry. There are now at the works—north and south—nearly 1000 tons of Aberdeen granite for the outer casing of the piers. Four steam stone crushers, each capable of crushing 80 to 90 tons per day, are at work preparing the whinstone for centering of the piers, and all along the line the greatest activity prevails in getting ready the preparatory operations required for this great structure.

THE INLAND SEA OF TUNIS.—At the recent lecture given by M. de Lesseps at the Sorbonne on the project of Captain Condarre for flooding the Shotts and creating an inland sea on the borders of Tunis and Algeria, the lecturer expressed his entire trust in the success of the operations from an engineering and sanitary point of view. The survey recently made by M. de Lesseps has satisfied him on these points. All the projectors require to begin with the work is the concession of useless lands which will form the shores of the lake. The evaporating power of the sun is less there than in the Red Sea, and M. de Lesseps does not anticipate that the waters will dry up. The bed of the cutting will be of sand to a very great depth, one boring of 73 meters at Tozeur still showing this kind of bottom. Politically speaking M. de Lesseps anticipates good results from the execution of the scheme, as the sea would form a frontier for Tunis and Algeria. A report of the recent survey has been communicated by the engineers of M. de Lesseps and M. Roudaire to the French Aademy of Sciences. The Oued Melah mouth, where the flooding canal will leave the Mediterranean, is so well covered at high tide as to form a natural port, especially when provided with jetties. The canal will be straight and its navigation easy; the anchorage in the sea will be of mud and sand free from rocks. The mean depth will be about 20 meters, or over ten fathoms. The soil of the north bank of the inland sea and along the canal from Galies to Biskra is similar to that of the most fertile parts of Algeria and Tunis, and only requires water to make it highly productive. The amelioration of the climate of the sea and the utilization of subterranean waters and wells which exist must, in the opinion of the engineers, be of great benefit to the surrounding country. The calcareous rocks at the entrance of the canal near Galies are considered an advantage rather than otherwise, as they will furnish building materials for the jetties and other constructions of the port, and sluices for filling the Shotts with water. The new route between the Shotts of Djerid and Rharsa avoids the rocks of Kriz, which are of considerable extent, and the altitude of its watershead is 12 meters less than that at Kriz. The canal proposed is 25 to 30 meters wide, but it is not expected that the current will widen it. The time required for excavating it is estimated at five years, and its cost is reckoned at 150 million francs. The report is signed by seven surveyors, among whom are M. A. Couvreux, contractor for public works, M. Emile Dollot, engineer of arts and manufactures, M. Leon Dru, engineer, and M. G. de Kersabiec, a naval lieutenant.

IRON AND STEEL NOTES.

ON THE EFFECT OF SULPHUR AND COPPER UPON STEEL.—Opinion is considerably divided as to the proportion of copper and sulphur that may be present in steel without injuring its working properties. Karsten, reproducing the opinion of the practical ironworkers of his time, states generally that copper makes iron red-short. Eggertz states that wrought iron with 0.5 per cent. of copper shows only traces of red-shortness. Stengel draws the following conclusions from the results of a series of experiments:—

1. Sulphur to the extent of 0.116 per cent., and 0.192 per cent. of silicon, without copper, renders iron and steel red-short and useless.

2. Red-shortness becomes apparent with 0.015 of sulphur, and 0.44 of copper per cent.

3. The deteriorating effect of sulphur is much more energetic than that of copper, 0.1 per cent. of the former being probably more injurious to the strength of iron than 0.75 per cent. of the latter.

According to Eggertz, steel made from an iron containing only 0.5 per cent. of copper is worthless.

In America, greater importance is attached to the absence of sulphur and copper in steel than in Germany, 0.15 to 2 per cent. of copper being considered as too high.

In order to obtain more definite information as to the influence of these elements, experiments were made by the author in 1875 at Bochum, by adding them both separately and together to the metal, in a converter containing 3-ton charges. Copper was added in the metallic form, and sulphur as sulphide of iron. The addition was always made before the charge was introduced, in order that the effect might be uniformly distributed through the mass. The ingots were rolled into rails, receiving the same reheating as those ordinarily made. A complete analysis was made in every case, in order to determine whether the red-shortness might not be due to other substances.

From the experiments the author concludes that the effect of copper in producing red-shortness has been over-estimated, as steel containing 0.862 per cent. was perfectly workable, and even in combination with sulphur it is not so very injurious if the latter is kept down below the limits at which it will produce red-

shortness alone. As an extreme limit to be tolerated, the author considers 0.15 to 0.16 per cent. of sulphur as likely to cause red-shortness, while 0.10 per cent. may be regarded as harmless. It may also be that, with softer and less manganiferous metal than No. 4, the injurious effect may be more marked, and in any case the less snlphur that is admitted the better.—*Abstracts of Inst. of Civil Engineers.*

BASIC BESSEMER STEEL.—We understood that plates made in the Bessemer converter by the Thomas Gilchrist process at Witkowitz, in Moravia, were recently sent to the Austrian Llyod's Registry, at the request of that authority, to be tested as to their suitability or otherwise for boilermaking and shipbuilding purposes: and that when they had been exhaustively tried, they were pronounced to have stood very satisfactorily all the tests required by the Llyod's committee. Such a result should be encouraging to those steel-makers who are preparing to make by this process plates for use in British boiler and ship yards. Authentic returns of the production of basic steel by the seventeen firms who are making it show that the annual out-turn is at the rate of 558,800 tons. In the six months ending with March the precise tonnage was 279,400. It was made to the extent of 57,911 tons by the one firm in England; 5,962 tons by the two firms in France; 12,786 tons by the one firm in Belgium; 152,479 tons by the nine firms in Germany; 37,476 tons by the three firms in Austria; and 12,786 tons by the one firm in Russia. The make by Messrs. Bolckow, Vaughan, and Co.—The one English firm at present working a basic plant—is, it will be seen, at the rate of 9,651 tons per month. This is considerably over three times the average make per month by individual German firms, the foregoing returns show, are not only the largest producers of this class of steel in the aggregate, but also the largest producers, per individual firm, of all the continental firms who have adopted the system.

RAILWAY NOTES.

IN the Cottran locomotive the driving axle, and those coupled with it, are provided with two pairs of wheels—one pair of the diameter best suited to the ordinary requirements of the road and the traffic, and the other pair—generally placed outside the larger—of a much smaller diameter, for running on supplementary rails when mounting heavy gradients. Thus, with a uniform number of piston strokes, and therefore revolutions of the wheels, the speed of the engine is reduced, with a corresponding gain of power, when there is harder work to be performed. This is not new as a proposal, and there are practical objections, such as obstacles lodging in the space between the inner and the outer rails, and capable of throwing the train off the track by contact with the clearing irons. Again, as the inner rails are necessarily lower than the outer, there is increased difficulty in forming level crossings; but this objection is of less weight than the former, as level crossings are a fruitful source of accident, and are being abolished by the Board of Trade wherever possible. The supplementary rails require to be laid very accurately, and tapered off at each end, so as to avoid any shock to the train when entering upon and leaving them.

THE following is the estimated railway mileage of the world, January 1st, 1883: United States, 113,000 miles; Europe, 109,000; Asia, 8,000; South America, 7,000; Canada, 8,500; Australia, 3,200; Africa, 2,200; Mexico, 2,100. Grand total, 253,000 miles. These figures are not claimed to be exact. It is absolutely impossible to obtain official returns for the same period within a year or two after date, and so it is necessary to use the latest available statement, and add the probable increase since that time.

IT is announced that the whole of the Centra Bengal Railway will be open for traffic by the end of the year, and that good progress is being made by the Bengal and Northwestern Railway. The earthwork of the entire system of the latter line will be finished early in June, and the greater portion of the brickwork. The *Times* Calcutta correspondent says it is also probable that 140 miles—from Sonepore to Goruckpore—will be ready to be opened next March, and perhaps even the entire line.

THE foreign press has been devoting attention to the scheme of a city railway for Paris, the construction of which has doubtless been suggested by the success which has attended a like project at Berlin. Preliminary steps have been taken towards obtaining the necessary powers from the French Legislature. According to the plans now drawn up, the railway will be in two parts, one extending from the Lyons terminus to the Arc de Triomphe, and the other from Montmartre to Montrouge. It is also proposed to construct ten subsidiary lines, uniting at the new post-office. According to the calculations of the commission which has been examining the project, the two main portions of the line would be completed within three years of the necessary powers being obtained. A low scale of fares is projected, and the scheme includes the utilization of existing omnibus and tramway routes in correspondence with the new railway system.

ORDNANCE AND NAVAL.

COMPOUND ARMOR PLATES AT SPEZIA.—The following conclusions from the results of further experiments at Spezia with pieces of the steel and compound plates fired at last autumn with the 100-ton gun are translated from the *Revista Marittima*, May, 1883:

From the experiments made at Spezia in March last against fragments of Schneider 48 cm. armor plates, it might be concluded that the metal behaves under test in an analogous manner to wrought iron, which at first was generally adopted for armor. The resistance to penetration in these plates is gradual, and in the metal which surrounds the point of impact there almost always appears a swelling with a versed sine in proportion to the quantity of metal of the projectile which has penetrated.

Moreover, the cracks which result from the shot all present a radial aspect due to the wedging action of the projectile, and these cracks, in the case of great penetration, develope also in the direction of the axis of the projectile, and result in a force tending to open the plate in the direction of the point struck The penetrations obtained in the trials are somewhat remarkable, although inferior to those which would have happened to rolled iron plates, for which, according to the Muggiano formula, should have shown the guns of 15 and 25 No. 1, a result between 23 and 24 and 32 and 33 cm. respectively.

It is true, however, when treating of fragments already damaged by previous shots from the 45 cm. gun, and struck at times at points presenting a great deficiency of resistance, either through want of support or from pre-existing cracks, such blows should not be reckoned; but we must consider that other shots have been fired against blocks in a good state of resistance, and of such relative sizes as to retain the proportion of weight between the 45 cm. projectile and the entire plate as tested in November last. In such a case the shot may be said to be sufficiently significant. At any rate, the continual occurrence of such phenomena, and their resemblance to those at Ochta, Gavre, and at Shoeburyness, give much weight to the information now collected. With these premises the experiments indicate that in firing against Schneider plates, projectiles of 15 cm. had about an average penetration of 94 mm., and projectiles of 25 cm., No. 1, a penetration of 163 mm. The behavior of the compound is very different from that of the Schneider plates; the hardness and the special tenacity of the steel-faced stratum tends to produce the breaking up of the most resisting projectiles, so that the resistance to the shot, instead of being gradual, may be constdered almost instantaneous.

The penetration is, therefore, much less than in the Schneider plates, and if we omit the blows upon points already much weakened, and by way of compensation those made with experimental projectiles, which were too weak in proportion to their energy, the average penetration is found to be 58 mm. for the 15a R.C. gun, and 74 mm. for the 25 gun, or 27 and 50 per cent. respectively less than in Schneider's. Nor do these figures fully represent the advantages as to penetration that may be expected from compound plates, since there is still included in the calculation the shot with the 25 gun against the Cammell fragment, in which, besides being excessively weakened, there was at the point struck a diminution of almost 4 cm. in the thickness of steel.

Excluding the result of the 25 cm. No. 1 gun, the average penetration against the composite plates was barely 58 mm., or the same as with the 15 cm. gun. This is remarkable, since it seems to indicate that in the composite plates the penetration remains always almost *nil*, whatever be the caliber of the projectile. The superiority of resistance of the compound plates depends certainly upon the high degree of hardness of their face, and it would appear it is not yet known how this can be given to hammered armor. But this excessive hardness would become a defect if there was not a stratum of rolled iron underneath. In fact with plates of one metal only, whenever a very hard quality is chosen the penetration of the projectile is diminished; but, on the other hand, most serious consequences arise by having too brittle armor. If, on the other hand, the metal is soft, much greater local effects are produced. For the plate of one metal only, the question is therefore reduced to finding a maximum limit of hardness compatible with a structure which is not brittle. The data that we have up to the present time are not sufficient for judging whether this limit was arrived at with the experimental Schneider plates, but we may assume with some foundation that it was not far off, since in all the blows struck it was remarked that the plates continued to crack for some minutes after the shots, an internal crackling being heard, and at intervals those metallic sounds which denote the process of separation of the metal.

The cracks in the case of composite differ considerably from those of the Schneider armor. Besides those in radial directions there were sometimes circular cracks, having their center at the point of impact analogous to those produced in a vitreous mass when struck. The only case in which a compound plate cracked completely was when, being imperfectly supported, it was subjected to a powerful bending force. One last remark seems necessary as to the bruising effects of firing upon armor plates of varying degrees of hardness. The method of fastening the plate contributes in no slight degree to modify the resistance that it presents to the effects of bruising. A plate of hard metal supported on a yielding backing is under very inferior conditions for resisting the force of the projectile to those of a plate of some non-ductile metal fixed in the same way. The former when not fastened in a rigid manner, if struck by a projectile, will be subject to a bending force tending to its fracture, and proportional to the distance between the point of impact and the edge of the plate. In the case of armor plates of large dimensions not fixed in rigidly, whenever the blow takes place towards the center the force we have mentioned is very considerable. This is why the Experimental Commission has wisely proposed a blow in the center as a test for the reception of the compound plate. With a more malleable metal the reaction of the plate upon a pliable backing would, to an extent, do away with the bruising power of the blow. This difference in the behavior of plates of different hardness depending upon the special manner in which they are fixed, has no importance for plates intended for ships, since in this case the system of support may be considered as perfectly rigid, and therefore absolutely favorable to compound armor.

BOOK NOTICES.

A Visit to Ceylon. By Ernst Haeckel. Translated by Clara Bell, Boston: S. E. Cassino and Co.

Although this is the narrative of an eminent

naturalist it is designed to interest and even charm the non-scientific lover of travel. No traveler is so alive to all external influences as the true naturalist; so the habits of the people, their costumes, the climate, the incidents of every-day life, are as surely noted as the objects that demand scientific classification.

The narrative is, throughout, sprightly, thoroughly descriptive, and altogether agreeable.

HAND-SAWS: THEIR USE, CARE, AND ABUSE. By Fred T. Hodgson, New York: Industrial Publication Co.

Many people use hand-saws. Nearly as many misuse them. This little book is written, not in the interest of saw manufacturers, but of that larger class, the amateur artisans, whose time, patience, and money he thinks should be saved.

Chapter I, on the history of saws, is full of interest, and the subsequent chapters on selection and use of saws are full of valuable information.

PLUMBING: A Text-book to the Practice of the Art or Craft of the Plumber, with supplementary chapters upon House Drainage. By William Paton Buchan. Fourth Edition. London: Crosby Lockwood & Co.

The text-book for the practical plumber under notice has grown out of a series of articles in the *Building News*, which were written in 1871. These articles had grown into a book in 1876, a second edition of which appeared in 1879, and now a fourth edition has been issued. The different forms of sanitary appliances are fully illustrated, and special attention has been paid in this edition to the subject of ventilation.

A RUDIMENTARY TREATISE ON CLOCKS, WATCHES, AND BELLS. By Sir Edmund Beckett, Bart. Seventh Edition. London: Crosby Lockwood & Co.

This is practically a ninth edition of Sir Edmund Beckett's work on clocks and watches, for the articles written by the author as the eighth and ninth editions of the *Encyclopædia Britannica* were abridgements of this book. The portion devoted to clocks is practical, and intended to help those who wish to make, or direct the making of their own clocks of superior character; but as an amateur is not likely to make a watch, the part on watch-making deals more with principles than with working details. A list of the great bells of Europe, with date, diameter, and weight, is given at the end of the book.—*Jour. Society of Arts.*

MISCELLANEOUS.

A FIRM in Paris has patented an invention for the instantaneous formation of steam, which permits of its use at once in the cylinder of the engine. A pump sends the required quantity of liquid between two plate surfaces, which are heated, and between which there is only a capillary space. The liquid spreading in a thin layer evaporates at once, without going into the so-called spheroidal state, and this steam acts in the cylinder as fresh formed steam. The speed of the pump is regulated by the engine, the pump being connected with the shaft of the engine.

A NICE opportunity, which should not be lost, of constructing a subway for gas, water, and electric mains, is offered by the construction of the Inner Circle Completion Railway along Cannon street. Along the whole length of this street a heading is made which is above the tunnel, and which will be filled in with earth unless the excavation is utilized for the construction of a subway. This might be done at a comparatively small expense, and thus save for ever afterwards the constantly recurring expense, and what is worse, street obstruction, by the operation of the gas, water, and electric companies.

THE MANUFACTURE OF CAMPHOR IN JAPAN.—The camphor tree is very widely distributed in Japan, being equally common on the three islands Niphon, Kinshin, and Sikok: but it thrives best in the southern portion of the kingdom, namely, in the provinces of Tosa and Sikok. The sea coast, with its mild, damp air, agrees with it best, and hence the chief production of camphor is in these provinces.

Dr. A. von Roretz, of Ottanyama, Japan, states that the only tree which yields the commercial camphor of Japan and Formosa is the *laurus camphoratus*, which the natives call *tsunoki*. Camphor is collected the whole year through, but the best results are obtained in winter. When the camphor collectors find a spot with several camphor trees in the vicinity, they migrate thither, build a hut to live in, and construct a furnace for making the crude camphor. When that place is exhausted, the hut is torn down and carried to another place. The method observed in obtaining camphor is very simple. The workmen select a tree, and with a hollow-ground short-handled instrument begin to chop off regular chips. As soon as the huge tree falls, the trunk, large roots, and branches are chopped up in the same way, and the chips carried to the furnace in baskets. The furnaces are mostly built on the side of a hill near a stream of water, and serve for the wet distillation of the chips. The furnace is of very simple construction. A small circular foundation A is built of stone, and upon this is placed a shallow iron pan F 2 ft. in diameter, covered with a perforated cover E luted on with clay. This cover forms the bottom of a cylindrical vessel B 40 in. high, and tapering to 18 in. at the top. Near the bottom of this vessel is a square opening D, which can be tightly closed with a board. The whole vessel is covered with a thick coating of clay C, held in place by strips of bamboo. The cover of this vessel G, which is also luted on with clay, has an opening K closed with a plug. Passing through the side of the vessel near the top is a bamboo tube L leading to the condenser H. This condenser is merely a quadrangular box, open below and divided up by four partitions into five compartments communicating with each other. The open side of this box dips into water and is kept cool by water drizzling over it.